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TAPE RECORDER BELT STUDY

Final Report

On

Fatigue Life of Seamless Polyester and Polyimide Film Belts

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OUTLINE OF TEST PROGRAM

The three aims of this program -

1. Determine fatigue lives under conditions susceptible to statistical treatment and pertinent to use in mechanical drive systems.
2. Determine significant effects of fabrication procedures and some environmental conditions on fatigue life.
3. Evaluate the application of the metal fatigue life computation and devise a method valid for seamless plastic belts -

are accomplished in 3 phases -

A. Determine and correlate fatigue lives of polyester belts to develop a fatigue life curve.

B. Test and correlate effects of environmental and fabrication conditions.

C. Evaluate polyimide belts using the techniques of Phase A with a small number of samples.

The work itself is divided into two classes:

1. Hardware for fabrication and test.
2. Statistics of test methods and data reduction.

Each of these major points is discussed and conclusions are drawn in the final report.

SUMMARY

Some of the terms used in this report may not be common to the readers' vocabulary, but are used in the interest of eliminating ambiguities. The glossary of terms in the back should be a preface to comprehensive reading of this report.

This test is concerned with seamless belts of almost perfectly cylindrical shape, formed from a sheet of plastic film. Belt dimensions may range in length from 1" to 105", in width from .04" to 1.75", and in several thicknesses from .0005" to .0145", of which selected sample sizes were tested.

Seamless belts, fabricated from polyester and polyimide film, were tested for fatigue life and the test results analyzed to provide more adequate data for reliability analysis than was previously available. The test procedures and the equipment used to determine the fatigue life are described. There is also a description of the methods and equipment used to fabricate the belts. Each test was discontinued if a belt survived 10^8 cycles. Fatigue failures resulted in broken belts before this limit and are described in the report.

A method for computing the fatigue life of these belts was deduced from data provided by an earlier report by Licht & White (1) on polyester film belts. This method is described but was found to be unsuitable under some loading conditions. A variation of the previous method was developed and it provides an adequate fit over a range of loadings which is greater than will

be encountered in practical applications.

Two phases (A, C) of the test program were devoted to tests to redetermine the fatigue life curve; Phase A being an extensive test of polyester film, and Phase C being an abbreviated evaluation of polyimide film. A relationship between the endurance limit and the bias of the fluctuating load was developed for each material tested. The fatigue data are presented as a series of fatigue curves, each representing a given survival level at a known confidence level. The stress range scale was normalized and named the stress ratio. The probability of survival ("reliability") is also deduced from the same data.

The second phase (B) of the test program provided data on the effects on polyester belt fatigue life, of fabrication and environmental variables. The results of this test phase indicate that the best fatigue life is achieved when thin, narrow belts are used and that there is a strong interaction between heat treat time and length-to-width ratio.

The materials tested in this program were represented by DuPont Mylar for polyester and DuPont Kapton for polyimide.

The report is followed by a Bibliography, a Glossary of Terms, and an Appendix containing the test data and fatigue life work sheets with calculation procedure.

PURPOSE

Seamless belts fabricated of plastic film have been used widely in tape recorders for space applications. This in turn has created a need for more adequate data to be used in reliability analysis. A test program was undertaken to develop this data in respect to the fatigue life of these materials. This program had three aims: (1) to determine the fatigue lives under conditions susceptible to statistical treatment; (2) to determine whether any of the fabrication and some of the environmental conditions have a significant effect on the fatigue life; (3) to determine whether a fatigue life computation method adapted from that used for metals was valid and if not, to devise a valid method.

The first aim of this test program is designed to provide data in a form which permits standard statistical analytical techniques to be applied. The application of these techniques makes it possible to determine the fatigue curve for a given survival level, and the confidence level of the curve. Additionally, the test data provides information about the behavior of the endurance limit stress range as a function of load bias.

In other literature, fatigue data are usually presented as a curve showing the relationship between a variable stress (stress range) and the corresponding life (in cycles of stress.) The reported variable stress is either a fully reversed stress or a unidirectional stress. It is "understood" in the field that this curve represents the life that the material should exceed, at the level of stress range selected. That is, the curve represents the lower envelope of the lives. Occasionally, a pair of curves is presented which represent the upper and lower limits of the lives to be expected; some of these show the actual test points in a scatter plot. All are usually plotted by visual estimation. The survival level and its associated confidence level for a given curve are not known, and the user does not have the time or the facilities to determine them. It should be remarked that only recently has the need for extreme reliability and the high cost of premature failure required more detailed information about the fatigue curve and its associated confidence level.

The fatigue stresses which occur in practice are frequently unsymmetrical, and there are several stress combination diagrams which have been used to obtain an equivalent stress range with which to enter the fatigue curve. These diagrams give different results, and the user is forced to assume that one or the other is representative of the material in question. The effect of load bias on fatigue life is equal in importance to the effect of stress range, and both must be used in conjunction to determine position on the fatigue life curve.

The second aim of this test program is to evaluate the effect of a number of fabrication and environmental factors. The fabrication of these plastic film belts requires that the material be subjected to strains which approach the ultimate value, and the material is subjected to heat treatment at an elevated temperature. It is not known whether any of these factors have a deleterious effect upon the fatigue life. In use, these belts are sometimes operated in environments which change the physical properties significantly. The net effect of these changes in the properties on the fatigue life is not known. This test program determines which of these factors has a statistically significant effect upon the fatigue life of these belts. This is done over a wide range of values of the several factors to determine whether a significant change in fatigue life results.

The third aim of this test program is to determine the validity of computing fatigue life under combined stresses using methods similar to those used for metals.

The report on polyester film belts by Licht & White (1) included a fatigue life curve. These tests were all run at one level of installed stress and with various ratios of pulley diameter to belt thickness and the curve is not directly usable for other installed stresses. The report did not state explicitly which thicknesses were tested, nor what the actual lives were.

Previous to this program, an attempt was made to generalize the Licht & White data by adapting a stress combination diagram which is used for metals. This method of life computation was checked against reports of our premature failures and showed a good correlation between predicted and actual lives. Some belts did fail much sooner than this computation predicted. This was "explained" as resulting from damage occuring during installation of the belt. The above data was obtained with unidirectional bending on a two-pulley system; however, belts are frequently applied in a serpentine path, a loading pattern bias which is well outside of the range tested. It was felt that experimental data in this area was needed, and if the above method of combining stresses were valid, the experimental data over this wider range of loading patterns would provide verification of the method. If the results were anomalous, the above method would be shown to be invalid, and the data to develop a valid method would be available. In any case, the third aim of this test program would be met: namely, a valid method of calculating the life of plastic film belts would be developed with experimental confirmation.

METHODS USED IN THE PROGRAM

The methods employed in this test program to determine the fatigue life characteristics of the plastic film belts can be divided into two classes: first, the hardware with which to fabricate and test the fatigue lives of these belts; second, the statistical approach and techniques, to determine the test conditions to be used, and to obtain the maximum amount of useful information from a limited testing program. These methods are detailed in separate sections on Fabrication, Testing, and Analysis and Statistics.

The hardware for fabrication was determined by the choice of constant stress (loaded area of the belt maintained at constant psi), constant strain (belt stretched at constant speed) and drape forming (proprietary) methods for producing belts. Basic hardware for these methods was in use, but modifications for more exact control of variables was necessary. The testing hardware was determined by considering the normal belt applications and operating conditions, including stress range, crowned pulleys, and stress cycles/min.

The constant stress method proved difficult to control and was dropped entirely. (See Fabrication section.) Phase A belts were made by the constant strain method only; however, in Phase B, this method required some excessively long fabrication times (3 hours) and was discontinued. All Phase B and C belts were fabricated by the drape method. All belts were edge trimmed to eliminate the width variation as formed, which is considered to be an unacceptable variable in our applications.

The first and third aims of this test program were accomplished in Phases A and C. In these phases, all of the fabricating variables except belt thickness were held constant at typical values. Therefore, the only variables tested were material thickness and loading pattern. Each set of conditions was tested five times (five replications). The data was reduced and combined by the statistical techniques explained in the Analysis (pg. 34). These results were analyzed and reduced to produce the fatigue life curves and computation method of Appendix C (Pg. 68). A full explanation of the above is included in the Analysis (Pg. 34).

The second aim of this program was accomplished in Phase B by the simultaneous testing of eight variables, in polyester belts only, then correlating the results in terms of statistically significant effects on belt fatigue life. The proper combinations of conditions can be selected such that each variable can be treated, analytically, as though all the other variables had been held constant. Thus, eight variables can be tested in the same number of tests required for one variable. This method of selecting combinations of conditions is called factorial testing.

To minimize the influence of unknown or uncertain factors on test results, the tests were performed and belts made in a random sequence developed by assigning numbers to each and using a table of random numbers to decide the sequence.

CONCLUSIONS AND RECOMMENDATIONS

The results of the tests performed in Phases A and C as shown in Appendix A (Pg. 55) indicate that proper application of belts can result in long fatigue life (greater than 10^8 cycles). Tests were terminated at 10^8 cycles, and 16% of the tests ran this long.

Appendix C (Pg. 68) includes the material required to calculate fatigue life for polyester and polyimide film belts, and has been written so that it can be used without further reference to the text. The procedure is written to give some suggestion of the statistical implications of the numerical results without obscuring the mechanics of the calculation. Blank work sheets, Figures 13 and 14, can be reproduced as needed. Two typical sample problems are shown in Figures 19, 20, 21, and 22. The method illustrated in this Appendix is universally applicable since it is based on a range of loading conditions which is wider than would be encountered in practice and the confidence level of the result is known.

In developing the method for computing fatigue life in Phase A, an element of major importance was realized -- i.e., belt life is found to correlate with the stresses developed in the side in contact with the pulleys. This raises a question when a belt is being run through a pattern with both sides in contact during a cycle (serpentine). Which side should be considered in determining the fatigue life, as both sides may not have the same stresses? The results are found to correlate if the lives of both are calculated and the lowest of the two is taken as the belt life, which is reasonable, since failure of one side will precipitate complete failure. For application, the results of Phases A and C indicate the fatigue life of a plastic film belt to be determined by the minimum stress and the range of stresses each point experiences in an operation cycle. If all else is constant, decreasing the minimum stress or the stress range will result in an increased fatigue life.

In practical applications, a method used to decrease the minimum stress (i.e., increase belt thickness) will result in an increased bending stress as the belt passes over the pulleys, thus increasing the stress range. The fatigue life may be optimized by "trading off" stress range and minimum stress. A most effective way to increase fatigue life is to increase the pulley size. This decreases the stress range, the installed stress to transmit torque, and the stress due to torque transmission. In general, the "trade-off" technique must be used to optimize a whole system.

In Phase A, each of the test runs consisted of only five replications, as an economic compromise between number of replications and proper coverage of the range of load conditions. Consequently, the final curves do include a considerable allowance for sampling variation. A significant increase in the number of replications would offer two distinct advantages. The allowance for sampling variations could be reduced, and an increase in predicted life by a factor of two, but certainly less than three, could result. Of even greater value would be the extension of the minimum expected life prediction to a larger sample size (improved prediction of reliability). In the present test, the results of one failure in five have been extrapolated to predict one in ten. These comments are only partially applicable to the results for polyimide film (Phase C). There were only five sets of test runs (5 replications each) made over a relatively narrow range of load conditions with this material. In this range, the endurance limit stress range was 35% higher than for polyester film. The standard deviation and sampling variation of both materials were so nearly equal that it was felt that the behavior of the polyimide film could be extrapolated as a constant ratio from that of the polyester film.

It has been determined that this 35% increase in endurance limit stress range is reflected as a fivefold or more increase in fatigue life, on direct substitution of polyimide data into the life calculations. Consequently, it is felt that a more extensive test program for polyimide film would be justified. Further testing of polyimide film should be considered as fundamental information and not as a ramification of this report. As such it should include an intensive stress study analogous to Phase A and a factorial test analogous to Phase B. This latter test should consider the variables: thickness, width, length, elongation (of inner edge), forming time, forming temperature, cycling rate, operating temperature. This factorial test would ideally precede the other portions of the test program to obtain the best results, since Polyimide film is relatively unfamiliar.

In Table 3 (Pg. 65), the results of the analysis of Phase B, considering the variables individually, are tabulated as variance ratios. These ratio values, considered as a statistical rating of relative importance to fatigue life, can be separated into three groups. Two variables have ratios exceeding five; five variables have ratios appreciably less than one; and the eighth variable has an intermediate value of 1.4. Interestingly enough, this intermediate value is known from Phase A to be significant. Based on the more extensive tests of Phase A, we would expect a 58% reduction in fatigue life at the higher stress ratio, while this test shows only 22% reduction. This sort of difference is not unexpected with data which have such a large variability. This variable (stress ratio) can be used as a criterion for judging the other tests.

The two variables which show definite significance are the material thickness and the length-to-width ratio. Here the length-to-width ratio implies width alone, since the length was held fixed and elongation was not a significant variable*. Both an increase in width and an increase in thickness effect a reduction in life. This is consistent with the current theory that the strengths at points are "normally" distributed and, therefore, the weakest point in a large sample will probably be weaker than the weakest in a smaller sample.

It would appear that, other things being equal, the minimum cross-sectional area required will result in improved fatigue life, and in this respect, unnecessary material would be a detriment. There is a strong implication that the volume of stressed material is the significant factor and that the life may also be related inversely to length.

(*statement to be used again)

The effect of length-to-width ratio and thickness should be further investigated to define clearly the effect of these variables which are implicated so strongly by the statistics. The length-to-width ratio should be investigated to separate or correlate the effects of length or width alone, and all of these belt volume elements should be more completely tested to provide a guideline for optimization of fatigue life (in the present study only the stresses are considered) and to more closely approximate an exact life prediction.

No significant difference in fatigue life between the constant strain belts of Phase A and the drape formed belts of Phase B was noted. On visual examination, the scatter of Phase A points was compatible with the scatter of Phase B points, although the conclusion was not statistically checked due to the error in the original method of calculation, which resulted in different stress levels between Phases A and B.

Examination of the interactions in Table 3 (Pg. 65) shows four interactions which are clearly significant and two more which are probably significant. Two of the clearly significant interactions and one which is probably significant involve thickness as one of the interacting variables, and the length-width ratio (*width) is involved in the remainder with a clear interaction between these two variables. The interaction between the thickness and width is a strong indication that the cross-sectional area (or possibly volume of stressed material) is involved inversely with fatigue life. The detrimental effect of thickness disappears at 200°F and is reduced at a higher stress ratio. The detrimental effect of length-to-width ratio is effectively negated by an increased heat treat time. The detrimental effect of increased length-to-width ratio almost exactly matches the pattern of thickness against stress ratio. In both of these last interactions the magnitude of change indicated is appreciable (50%), but the variability of the data is large enough so that it can only be said that the effect is probably significant. The interaction between length-to-width ratio and cycling rate appears to be statistically significant; however, the deviation is so small compared to the effect of length-to-width ratio alone that it is felt that this is probably not really significant. In any case, the residual effect is so small it can be neglected in life predictions.

For polyester there does appear to be a strong interaction between heat treat time and length-to-width ratio, and it would be desirable to test further to determine the optimum time for each combination. Further testing of the relationship between thickness and operating temperature would be useful in providing test data at elevated temperatures and would provide insight into the interaction.

The confidence levels quoted for the curves of this report are really quite conservative. At each stage the curves were taken through the lowest of the low points. The author did not have the time or mathematical sophistication to derive a procedure for plotting a faired curve, at a given confidence level, through the test points. If this procedure could be derived, a longer life prediction at the given confidence level would result.

In summary, it appears the best fatigue life in polyester belts will result from optimization of belt stresses, volume elements, heat treat time, and operating temperature. At the present stage, only the stress variable has been thoroughly tested. Further testing of the remaining variables is desirable. Considering the results of the abbreviated testing of polyimide material, it is desirable to perform an extensive test to determine its full value.

BELT FABRICATION

The fabrication of belts for this test program was done on the machine shown in Figures 1, 2, 3 (Pg. 50) and by a proprietary drape-forming process. All belts of Phase A were formed by one method (constant strain). It was desired, in Phase B, to test the effect of different methods of fabrication on fatigue life, but technical difficulties required elimination of both planned methods and the comparison between them in this phase, and the drape forming method was substituted and used for all belts in Phase B. Experience with polyimide indicated this last method to be the suitable one for Phase C also, and it was used exclusively.

Two different mechanisms were incorporated in this machine to separate the spindles and thereby stretch the belt blank into a flat belt. One mechanism is a lead screw driven by a variable speed drive (see Fig. 2). This enabled us to set the rate of strain (which was held constant for any one class of belt) at the desired value. Belts formed by this method are called "constant strain" belts. In our experience belts formed in this way perform satisfactorily; however, a theory of belt forming indicates that this may be harmful. It assumes that there is a very uncertain relationship between the rate of strain and the rate of yielding, and the material may be carried too close to the Ultimate Tensile Strength of the material, in spots, with subsequent hidden damage. This theory concludes that the stress should be held at a constant level as close to the yield point as practical.

The second feed mechanism was built into the belt fabricating machine to implement this "constant stress" method. This mechanism (shown in Fig. 3) consists of a cable and a push-rod assembly which applies a separating force between the forming spindles equal to the load on the end of the cable.

All spindles are crowned by a 2° taper on both sides of a 3/4" flat, with blended corners, in order to maintain the belt on the spindles. In use, an oven is positioned to surround the forming spindles as shown in Fig. 1. The remaining feature of the belt fabricating machine is the cooling system. The forming spindles are heated during the entire forming and heat-setting period and must be cooled before the belt is removed. Both spindles are gun drilled nearly the full length, and a small centrifugal pump unit, located on the floor beneath the machine, pumps cooling oil through small stationary tubes to the far end of the spindles. The cooling oil then flows back freely to the end of the spindle, is collected in a banjo-type fitting, and drains back to the pump unit. This cooling system can be seen in Figures 2 and 3.

The constant stress method was abandoned after several difficulties were encountered with the technique. The first test belts fabricated by the constant stress method revealed a design oversight. The basic assumption had been that a constant load on the cable of the feed mechanism would provide a constant stress on the belt as it was formed. Two things became apparent when the first trial was made. First, the load carrying area of the belt blank varied from zero at the start to a maximum as the flat blank was stretched into the cylindrical form. After the blank was pulled into cylindrical form, continued elongation required still higher loads. As a first attempt to solve this problem, several belts were fabricated by starting with a low weight and adding additional weights when the rate of yielding became imperceptible. Most of these belts did stretch to their full elongation, but the variation in width was excessive. As the entire assembly was quite springy, it was felt that this excessive width variation might be caused by load peaks as the weights were added. The cable loading system was then modified to incorporate a bell crank so that the load on the belt varied as a sine function of the elongation. A few trials were sufficient to show that the load variation was still inadequate. A load which was just right at the start of forming would not be enough to finish pulling the blank out to final length. The loading system was further modified to increase the load variation by mounting the cable drum off center. The belts could be formed with this setup; however, the width variation was still excessive (about plus or minus twenty percent). This large variation in width was incompatible with proper control of the variables tested. The time allotted for the test program was running out and the prospect of obtaining adequate uniformity seemed dim enough to justify a change in the method of making belts with a concomitant change in the planned fractional factorial design of Phase B.

The machine as originally conceived and built for the constant strain method, had a fixed feed rate which was selected as the average used for the hand feed in our usual belt fabrication. Early experience with this machine indicated that the usable feed rate varied inversely with the elongation of the inner edge of the belt blank. A variable speed drive was interposed in the constant strain feed mechanism and held at a fixed speed for any one class of belt. After this was done, there were no further difficulties experienced with this method, and all Phase A belts were formed by this method. In Phase B, the most severe forming (90% elongation of the inner edge) required three hours to form the belts and did result in a useful yield. However, this fabrication time was excessive in view of the allotted test time. It was decided that the results of Phase A (using this method of forming) could be applied to a conclusion in Phase B concerning the effect of forming method on fatigue life. Therefore, the constant strain method was discontinued in Phase B.

The third method of forming belts, which was substituted in Phase B, can be called a drape forming process. In this process, a forming mandrel is preheated to the desired temperature and a sheet of plastic film with a hole of the proper size is forced over the mandrel. This extrudes a short tubular section which is "set" by a timed soak at the forming temperature. During the forming process and the subsequent heat soak, the internal orientation of the film is changed permanently. At the end of the soak period, the mandrel and formed blank are quenched in water, and the blank is removed from the mandrel. For this test program, the mandrel was heated in a small standard laboratory oven. This method was used in Phase B exclusively for reasons mentioned previously, and in Phase C for convenience and ease of control.

It was decided that the width variation as formed was not compatible with control of the other variables; therefore, after forming, all belts were trimmed to width by shaving the edges: this was accomplished by holding a razor blade against the edge of the belt while the belt was being run between two pulleys. An older method, grinding with an abrasive wheel, was tried and rejected as it was slower and more difficult to control.

These fabrication processes result in a high yield of cylindrical belts with close dimensional tolerances:

$\pm 0.25\%$ in length

$\pm 0.005\%$ in width

$\pm 15\%$ in thickness

No physical measure is taken of the tolerance on cylindrical shape, but any convergence of the sides in acceptable belts is difficult, if not impossible, to discern visually.

BELT TESTERS

Figure 4 (Pg. 53) illustrates one of the testers used in this program. The motor is a split-phase synchronous motor and can be equipped with either of two sizes of drive pulley to vary the stress cycling rate. At the other end of the tester, the idler pulley assembly can be seen. These two pulleys were crowned by cutting a 2-inch radius arc across their width to provide belt guidance. The idler pulley assembly incorporates several features. The assembly can be adjusted in and out to set belt tension. The pulley and bearings are pivoted in the frame so that a normally open switch is held closed by belt tension. An adjustable stop limits the motion of the pulley assembly and permits adjustment for proper belt tracking. The switch, when closed, runs the motor and an elapsed time meter connected in parallel with the motor. When the belt breaks the switch opens, stopping the motor and the elapsed time meter and turns on a neon lamp to provide a visual indication that the tester has stopped.

To obtain the maximum testing capacity with a fixed material budget, it was decided to equip each unit with eight test spindle assemblies of variable spindle diameter to provide the belt stress pattern. Eight bearing capsules are mounted on the base of the tester between the motor and idler pulley. The capsules are adjustable in a direction transverse to the centerline of the tester. The capsules contain two R4 bearings and can accept any diameter test spindle.

The testers were designed to be suitable for a wide range of tests. The test spindles were obtained in several diameters to provide different amounts of bending stress in the belts and were cylindrical, without any crowning. The belts can be threaded outside all test spindles or on alternate sides (serpentine) of the test spindles to obtain various amounts of stress range. The installed stress can be varied over a wide range for various degress of bias. Sixteen testers were built for operation at room temperature. Two additional testers were modified for operation at elevated temperatures. The modification merely provided for remote mounting of the motor and a laboratory oven to provide the elevated temperature environment.

The test technician was provided with a data sheet (Pg. 54) for each test run. The data sheet spelled out the fabricating and test conditions to be used. The test conditions which were specified were the fabrication conditions, the operating temperature, the test spindle diameter, the motor pulley diameter, the threading pattern and the load on the idler pulley. The test spindle diameter, threading pattern and load on the pulley determine the severity of the fatigue loading. The motor pulley size determines the cycling rate. The test technician sets the test belt loosely in place and shifts the test spindles in or out to obtain about equal wrap (15°) on each test spindle and to leave the idler with enough adjustment to permit tensioning. The belt length was measured under a known tension before testing. The tension was applied with a fish scale and the idler pulley assembly locked in place. The reading of the elapsed time indicator was recorded at the start and end of each test. Once started, the test ran continuously until failure or until 10^8 stress cycles had been accumulated.

During the first tests it was found that the motors had excessive starting acceleration for the belts tested. The torque required to accelerate the test spindles exceeded the capacity of the belts which then slipped off the motor pulley. it was necessary to add a resistor in the starting winding to reduce the starting acceleration to an acceptable value. The smallest diameter test spindles (.100 inches) ran at a speed of 37,000 RPM and the capsule ran quite warm. An excessive number of bearings failed under these conditions. It was finally determined that the temperature rise upon start-up was causing the air inside the capsule to pump the oil out of the bearings. This was cured by drilling a breather hole in the side of each capsule. There were periods during the test program when the test technician could not fabricate belts fast enough to keep all the testers running. At other times he had to wait for a tester to be open. It does not appear that the optimum number of testers is very far from the number used.

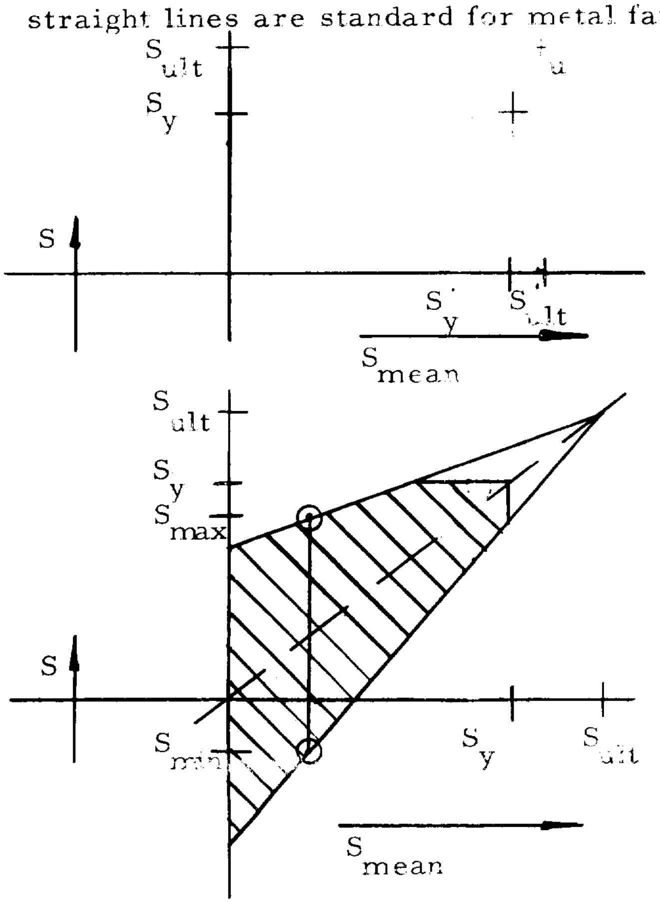
ANALYTICAL AND STATISTICAL TREATMENT

This test program is divided into three phases (A, B, C); of these, statistically A and C are treated alike, while B is treated by a different technique. The largest number of tests (Phase A) was performed to redetermine the fatigue life curve with known confidence levels and survival levels at given sets of operating conditions for polyester belts. This data was used to deduce and justify the method of calculating belt fatigue parameters in any general operating conditions. Phase C consisted of a similar although abbreviated test of polyimide belts. The test conditions were controlled at values commensurate with conditions we normally employ. The other part of the test program (Phase B) was designed to determine whether some of the fabricating or environmental variables had statistically significant effects upon fatigue life and also, if the effects were significant, to determine the magnitude and direction of change.

In a previous report on polyester film belts by Licht & White (1) a fatigue curve was presented. The test conditions for their test were a fixed value of installed stress (1600 psi), which is lower than normally used in practical application, and various levels of bending stress $\left(\frac{\text{Elastic Modulus x belt thickness}}{\text{bending diameter}}\right)$. The ordinate was plotted in terms of the bending stress at the smallest pulley; an auxiliary scale on the ordinate was plotted in terms of the pulley diameter to belt thickness ratio. Our first attempt to generalize this curve was made by the following procedure:

The maximum stress at the outermost fiber of the belt was calculated by adding the bending stress to the stress due to installed tension. The minimum stress in the outermost fiber was simply the installed stress. This was done at that point on their fatigue curve where the life was 10^7 cycles. This point on the curve was selected because the endurance limit stress range is frequently (and here) defined as the variable stress which will give a life of 10^7 cycles.

The maximum and minimum stresses determined above, and the yield and ultimate strengths of the material are combined in a graphical method for determination of the endurance limit vs. mean stress, $\left(\frac{S_{max} + S_{min}}{2}\right)$, curve as shown below. This method and the assumption that the curves are straight lines are standard for metal fatigue life computation (Pg. 84, Ref. 11.)

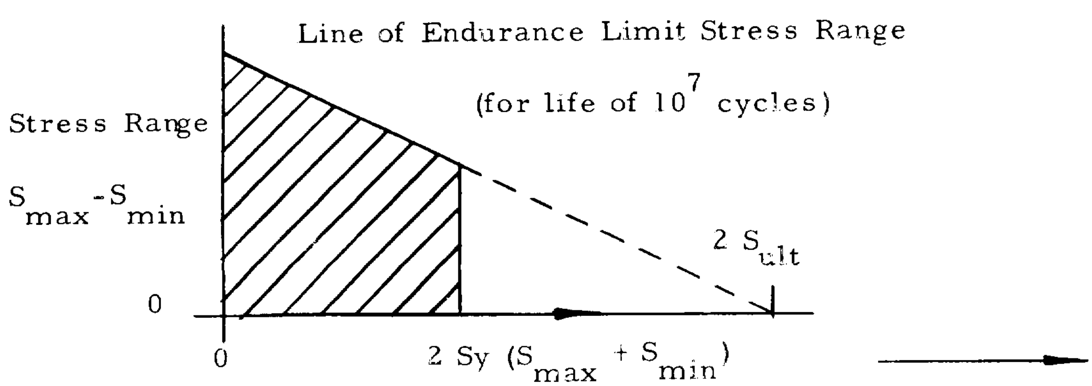


The diagram is drawn with the operating stress as the ordinate and mean stress as the abscissa. On these axes the ultimate (S_u) and yield (S_y) stress are marked and the corresponding points (U, Y) plotted in the plane.

Now for the particular operating stresses from above (10^7 cycles at S_{mean}) we plot S_{max} vs S_{mean} and S_{min} vs. S_{mean}. The line connecting these points covers the range of stresses encountered in this particular operation.

It seems evident that if the stress range were increased or decreased, the life would be inversely affected. "From analysis and experiment, the lines from U through these last two points to the ordinate determine the bounds inside which fatigue life can be expected to be at least the value chosen for endurance life (here 10^7 cycles). With stress ranging outside these bounds, fatigue life is expected to be less than the endurance life." (Pg. 84, Ref. 11). Normally, operation beyond the yield stress (S_y) is not desired; this determines the lines at Y as additional limits on the endurance diagram (hatched area). This diagram is extended into the other half plane in the same way.

This diagram is usually drawn from tests with fully reversed variable stress. ($S_{mean} = 0$). In this case, we have worked backward from a known load pattern to determine the diagram. Implicit in this diagram is the assumption that the endurance limit varies linearly with the mean of the maximum and minimum stresses. The above diagram was modified to simplify fatigue life computations. The stress range (maximum minus minimum stress) was plotted against mean stress in a second diagram shown below. The two diagrams are equivalent.



In this diagram the abscissa was rescaled and relabeled with the sum of the maximum and minimum stresses to save a division by two in every calculation. The diagram is cut off at the point where permanent yielding would occur. Every point on the Endurance Limit Stress Range line represents a set of values of stress range and mean stress which (it is supposed) gives an expected life of 10^7 cycles of stress. Operation in the hatched area will increase the expected fatigue life while operation out of it will decrease the expected fatigue life. As before, this diagram may be extended to the negative abscissa.

The bending stress scale of the curve in the report by Licht & White (1) was normalized and named stress ratio. The stress ratio was defined as the ratio of the actual stress range to the endurance limit stress range at the same mean stress (this definition was modified later as a consequence of the test data developed in this program). The procedure was then to determine the sum and difference of the maximum and minimum stresses. From the sum (mean stress times 2) enter the diagram above and determine the endurance limit stress range. Divide this value into the difference (stress range) to obtain the stress ratio. The normalized fatigue curve from Licht & White is then entered with the stress ratio to determine the predicted life. This curve however, is only as good as the test results which produced it; the values for S_u and S_y are not absolute but have some statistical uncertainty, the values of operating stress S_{max} and S_{min} will not always produce 10^7 stress cycles before failure. The curve becomes fully useful only when we know the limits of the many uncertainties involved in the curve determination. Statistically, the general argument on fatigue life reworded in in terms of this study, is that if the fatigue lives of individual points on the belts are even approximately normally distributed and all points on the belt are subjected to the same number of stress cycles, then testing the belt to failure is equivalent to determining the shortest lived point in a very large sample. If this process is repeated a large number of times, the failure times of the shortest lived points have a distribution of their own.

The above, when done mathematically, results in the expression used (Weibull Distribution). If the logarithm of life is plotted against units of standard deviation of the distribution function on a cumulative basis, and if the function fits the population, a straight line plot results. The first few completed test runs of this program were used to verify the validity of the Weibull Distribution as used here.

The values of this distribution function in terms of the several parameters are tabulated in Ref. 7 and in a somewhat less useful form in Ref. 2. Two methods for actual construction of Weibull Distribution Probability paper are shown in Appendix D (Pg. 85). The actual scale is shown in terms of percent failed, and is non-linear.

When a finite sample is tested, all of the samples will fail if the testing is continued. If a further sample is added to the original sample, it is known that the larger sample will probably show a wider range of values and that the larger the sample, the wider the range. The actual points in the original sample should be plotted where they would probably lie if an infinitely large number of tests had been run, and these tests were a part of this large number of tests.

This raises the question of what method should be used to plot the test points in a position where they will best describe the expected distribution of infinitely many test points. This requires some adjustment of the actual data for plotting purposes since a finite sample is being tested.

Here, the relative number of failures (portion failed) is modified; i.e., for the first failure of five replications, we would say 20 percent had failed, but for plotting purposes, this figure would be changed. Johnson (2) advocates the use of the median rank (the percentage failed which the test point is equally likely to exceed or not) as a plotting position; however, this value is dependent on the distribution function. Gumbel (4) advocates the mode (the most frequent occurrence of the value) as the plotting position. The mode is independent of the distribution function (4) and was used in this study as the plotting position. In terms of portion failed, the value of the mode of the mth failure in a sample of size n is $m/(n+1)$. As, for the example above, the life of the first failure of five replications would be plotted at $\frac{1}{5+1}$ (17 percent).

The analysis procedure is to arrange the n tests in order of increasing lives (rank) and then to plot each of these lives against its corresponding plotting position. Due to sampling variations the actual points will be scattered randomly about the straight line which would have resulted if n were infinitely large. This line is estimated by applying the method of least squares to the test points. This calculation provides the equation of the line which fits the test points with the smallest variation about the line and also provides the standard deviation of the test points about this line.

The next step in the calculations provides a prediction interval about the best fit line. The prediction interval, a band extending on both sides of the above straight line, is calculated to include a pre-selected percentage of all test points that would be expected under the same conditions. In this study the 90% prediction interval was selected. Then no more than 5% of future test points could be expected to have lives shorter than the lower limit of the prediction interval and no more than 5% of future test points could be expected to have lives which exceed the upper limit. Since our concern is with how long we can be sure the equipment will be operable, the lower limit of the prediction interval is the one of interest. We can then say, "at least ninety five times out of a hundred (a 95% confidence level), the life, when a given portion have failed, will exceed the value of the lower limit of the prediction interval of the same portion failed." Most of the remaining five times will be close to the lower limit. All this refers to test points obtained under the same conditions of stress range and bias.

In this study 95% confidence level was selected as a useful compromise between assurance and economy of testing. Phases A and C, which were determinations of fatigue curves for polyester and polyimide belts, were treated in this way.

The raw data points are also plotted on a stress ratio vs. life graph. The stress ratio ($\frac{\text{Stress Range}}{\text{Endurance Limit Stress Range}}$) had been calculated before conducting the test by using the original technique for the Endurance Limit Stress Range on Page 27. The points are then considered in relation to the established fatigue curve and are used to check the test progress and the coverage of the desired range.

Specific numerical details and the modified analysis are discussed and developed in the following section, Results and Analysis.

A great deal of work has been done to develop testing techniques for the determination of optimum conditions with several variables. It is possible to test the effect of each variable separately and then the effects of each combination of variables and then determine the optimum condition. This usually involves an impractically large number of tests. Mathematical analysis shows that the same accuracy and sensitivity can be achieved if all the variables are tested in the proper combinations of conditions at the same time in a much smaller number of tests. This testing method is called factorial testing and the set of combinations of conditions is a factorial design. In a factorial design every possible combination of variables is tested once. The entire design can be repeated as many times as necessary to measure with the desired sensitivity. When a block of tests has been completed according to a factorial design it is possible, by analytical techniques, to separate out the effect of each variable as though it had been the only variable, and the effect of each interaction between variables as though these variables were the only ones tested.

Still further economies in testing can be bought at the price of some loss in sensitivity by properly selecting a fraction of the tests from a full factorial design. This design is called a fractional factorial design. The effects of at least some of the variables are mixed up ("confounded" is the term used in this field) with the effects of some of the interactions. The experimenter is able, with certain restrictions, to select which variables are mixed with which interactions.

The design used in this study is a 1/8 fractional factorial design for eight variables at two levels. In this case three variables are mixed up with four factor interactions. This phase of the study is intended to determine which of the variables do have a significant effect and to determine the direction of change of the variable which would result in improved fatigue life. If significant effects are found, further testing would be required to determine the optimum values. This further testing would be restricted to those few variables which were found to be of interest. The cost in terms of sensitivity in this screening test is not considered of importance. A complete discussion of this type of testing is available in Reference 8.

RESULTS AND ANALYSIS

The test conditions used and the lives obtained are tabulated in Appendix A and B (Pg. 55). The data is also shown graphically in the form of scatter plots. Figure 6 shows the data for Phase A (polyester film) with the stress ratios calculated by the method developed as a result of the discrepancies shown in Figure 5. (Fig. 5 is based on the first, now obsolete, method.) Figure 7 shows the data for Phase C (polyimide film). Figures 15 and 16 in Appendix C (Pg. 68) show the relationship between the endurance limit stress range and the minimum stress in a stress cycle for polyester and polyimide film belts, respectively. Figure 17 is the working fatigue curve which was developed by this Study.

During the course of the testing program the belts which failed under test were spot examined. The visual appearance of the break followed one of three general patterns. There was no count made of the number of breaks which followed each pattern. One form of break is characterized by a fracture surface substantially square to the surface and the length of the belt with a small tit at one edge. The second general form of break is similar to the first, except that the break line would turn from square to an angle between 30° and 45° from the length of the belt, within the width of the belt, and then turn back square. The third general pattern showed the same shape as the second form except that delamination occurred in the area of the break. This delamination could extend over enough area so that the break lines on the two surfaces of the belt were offset up to one-quarter inch.

A typical run from Phase A is shown plotted on a Weibull Plot in Figure 8. The calculation of the best fit line and the confidence band are shown in Figure 9. The computation is covered in detail in Reference 3, Page 238. The interpretation of this plot is that the straight line is the best estimate, based on the sample of the line which would describe the relationship between the life obtained and the portion failed if an infinitely large number of belts had been run. There is, however, a known degree of uncertainty in the position and slope of the line due to the variation in the finite sample tested. A further uncertainty is introduced in any other sample which will be tested later. Therefore, all that can be said for a subsequent sample is that at least 90% (in this case) of the time the lives obtained at a given portion failed will lie within the prediction interval.

Example: Given - Ten test runs with five belts each, under the same conditions as those which produced the Weibull plot. Plot the live vs. % failed for these new runs on the same plot. Result: Of the ten points produced by the first failure in each run, at least nine will be expected to lie in the prediction interval. The same prediction is made for the second failure in each run, and so forth.

The 90% prediction interval was calculated so that we could say, "95% of the time the lower limit of the interval would be equalled or exceeded." Each run of five belts was treated in this way if at least three belts failed before 10^8 cycles. All tests were stopped at a life of 10^8 cycles of stress, and less than three failures in five resulted in discarding (for analytical purposes) the results because of the limited utility of the data.

The value of the slopes of the plot and the variability of the data about each line were tested by the use of Cochran's test for the homogenity of variances (Reference 3, Page 198). The slope of the Weibull plot is the standard deviation of the distribution function. Here the standard deviation is a measure of the spread in life of similar belts under similar conditions. Its square is the corresponding variance. Cochran's test for a group of variances consists of taking the ratio of the largest variance in the group to the sum of all the variances. and comparing this ratio to that which would be expected by random variation. This ratio is a function of the confidence level selected, the number of variances in the group and the sample size for each variance.

For the slopes of the plots the ratio is 0.1375 compared to the tabulated value of 0.185 for the variables involved here. The variances $(\text{slope})^2$ can be considered equal, and an average value of 1.437 of the variance determines the most probable value of the slope as 1.20.

The variance of the points about the line is obtained as a step in the method of least squares, and indicates the spread of points about the line. In the same manner as above, Cochran's test indicates that these variances can also be considered equal with a value of 0.391.

These results verify that one slope and one prediction interval can be used for all plots. The fatigue life curves for any two portions failed will have a constant ratio of lives. A similar test of the statistics of the polyimide film belts (Phase C) shows that here also the slope and prediction interval can each be considered common with values of 1.22 and 0.446 respectively.

The distributions of the two materials are very close to each other.

The raw data points are plotted along with the established fatigue life curve; the scatter plot shown in Figure 5 demonstrates the wide divergence between the lives obtained and the stress ratios calculated by our original method. (Pg 27) This discrepancy was very disconcerting. An examination of the test conditions of the runs which were very short or very long lived showed that the serpentine threading pattern was shorter lived than expected, at low stress ratios, and the outside threading pattern longer lived than expected, at high stress ratios. This clue indicated that an improved fit might result from the use of a different value of the modulus of elasticity. This was tried for several values of the tensile modulus and only slight improvement of the fit was noted. This approach seemed to be a dead end. Various other approaches were tried without result. In each case, a correlation plot was made between some calculated value and the corresponding value determined empirically.

It was finally determined that the apparent stress ratio (apparent is used in the sense of observed) was the pertinent value to be used in the correlation plots. The apparent stress ratio for each run was determined by finding the life for median survival (50% survival) at the lower limit of the prediction interval (from the Weibull plot for each run). Using this life value (cycles of stress), we find the value of the corresponding stress ratio on the established fatigue curve. This is the apparent stress ratio. The apparent endurance limit stress range for each run was then calculated by dividing the stress range for that run by the apparent stress ratio. Since the stress ratio is the ratio of the stress range to the endurance limit stress range a simple inversion explains this step.

These apparent endurance limits were then plotted against the mean stress for each run. This plot showed two groupings. Each group lay along a line and the two lines were parallel to each other. At this time a suggestion was found in Reference 9, Page 322, that the endurance limit might be correlated with the minimum stress in the cycle rather than the mean stress. This was found to be true and each of the groupings was found to lie closer about its average line. If you refer to Figure 10 and shift each of the circles about five inches to the right you will see the appearance of the plot at this stage. All the outside threading runs lay in one group and all the serpentine threading runs lay in the other group.

It would have been possible to set up two endurance limit stress range curves, one for each threading pattern. This would have been clumsy and unsatisfying. Some universally applicable method would be easier to use and would be used with more assurance. After some deliberation it was realized that both sides of a belt with a serpentine threading pattern (all the test spindles were one diameter in any one test) went through the same stress cycle but that the stress cycle was appreciably different on both sides of a belt with the outside threading pattern. The inside surface of the belt compresses when running on a pulley while the outside stretches. This changes the minimum stress in the cycle. The apparent endurance limit stress ranges were recalculated and replotted based on the stress cycle of the inside of the belt and using the original value of the tensile modulus of elasticity. This brought the two groups together as shown in Figure 10.

The effect of different values of the tensile modulus was found to be a shift of all the points along the direction of the trend line. This indicates that an exact value of the tensile modulus of elasticity is not necessary for an accurate calculation of stress ratio. The same value used in calculating the stress ranges for the curve must be used to enter the curve, but the only requirement is consistency. The curve for polyester film is based on a value of 750,000 psi for the tensile modulus.

A curve is drawn through the lowest of the points in the above plot and through the ultimate strength at a zero stress range to represent the relationship between the endurance limit stress range and the minimum stress. As a check for accuracy, the stress ratios of all the test runs were then recalculated using the new endurance limit stress range curve and using the new method. All the test points were replotted as shown in Figure 6. It will be seen that only four test points fall below the curve.

The fatigue curve was not changed, but it is now required that the curve represent the median failure rate with a 95% confidence level. In other words, 95% of the time at least one-half of a sample will survive at least as long as shown by the fatigue curve. Figure 6.

In retrospect, the resulting technique is:

#1. Assume the validity of existing fatigue life curve (shape and position).

#2. Define this curve to be that for a particular percentage failed, at a particular confidence level. (Here 50% failed, 95% confidence level).

#3 Make test runs to determine the actual life at confidence level and percentage failed from Weibull Plot.

#4. Determine the Apparent Stress Ratio from Nos. 1, 2, and 3.

#5. Calculate Apparent Endurance Limit Stress Range from #3 and known stress range.

#6. Construct a plot of #5 vs. minimum stress in the stress cycle.

After the median failure level curve has been established, the curves for other survival levels can be determined. It was previously shown that the slopes of all the cumulative failure plots could be considered equal and that the variances of the sample points about the best fit lines could also be considered equal. This implies that the fatigue curves for two survival levels will show a constant ratio at all stress ratios in the range tested. It is then possible to construct a Weibull Plot with the average slope and the average prediction interval and from this plot measure the ratio of lives between any two survival rates. This has been done for failure levels of 10% and 20% (really sample sizes of 10 and 5) at the 95% confidence level and also for the most probable value (50% confidence) of the median failure level. These curves can be seen in Figure 16. The solid curves are for a 95% confidence level for the sample sizes shown on the curve and the dashed curve (50% confidence level) can be used to predict median (1 out of 2) time to failure.

The data from Phase C, for polyimide film belts, was treated in the same way as above. The resultant scatter plot is shown in Figure 7 and the endurance limit stress range curve is shown in Figure 12. Use 590,000 psi for the tensile modulus for this curve. One obvious fact that appears is that the endurance limit curve is 35% higher than for polyester film belts in the range tested. This higher endurance limit and the lower tensile modulus of elasticity should result in an increase in life of five times or more upon a direct substitution.

It was shown earlier that the Weibull slope and sampling variability for polyimide and polyester film belts were equal. The same set of fatigue curves can then be used for both materials. It is only necessary to use the endurance limit stress range curve for the material being used. The polyimide film used in this phase of the test program was contributed by E. I. du Pont de Nemours and Company, Inc.

The computations involved in an analysis of variance for the fractional factorial design are very simple and straightforward, but also very laborious. They are completely covered in various standard references, such as Reference 8. In this study the computation was done by a standard computer program and the computer results are summarized in Table 3. The end result of the analysis is a series of variance ratios which compare the variability caused by a change in level of the variable or interaction between variables being considered and the random variability of the test data. This ratio is then compared to the value of a function known as the F-distribution.

The F-distribution is the maximum value the ratio could have, at a given confidence level, due to sampling variability. If the F value is exceeded the variable is said to have a (statistically) significant effect.

These comparisons are shown in Table 4 for the effects found to be significant. It should be noted that the value of the F-distribution is a function of the confidence level and the degrees of freedom (roughly sample size) of the numerator and denominator of the ratio under comparison.

In summary, the results of Phase A produced a few critical elements which must be clearly emphasized. In temporal order they are:

1. All Weibull slopes and prediction intervals are shown approximately equal.

2. Accept the established Fatigue Life curve, but only at 50% Failure, 95% confidence level.

3. Use of Fatigue Life curve and actual test life Weibull plots to produce Apparent Endurance Limit Stress Range values. (This instead of the original method of Page 26).

4. Establishment of minimum stress rather than mean stress as the reference (bias) for plotting these Apparent Endurance Limit Stress Range values to produce the curve for new predictions.

5. Change of the belt reference layer from that at the outside of the bend at the pulley to that layer at the inside.

The results of Phase C are consistent with the techniques of Phase A and are interpreted in the Conclusions and Results section, as are the results of Phase A and B.

GLOSSARY OF TERMS FROM THIS REPORT

To facilitate use, the words are listed in alphabetical order and referenced to a numerical sequence in logical order.

Average	19	Mean Stress	3
Belt Length	15	Median	20
Bending Stress	4	Mode	21
Bias	12	Population	17
Confidence Band	27	Portion Failed	29
Confidence Level	26	Prediction Interval	27
Correlation Plot	32	Probability Distribution Function	25
Cumulative Failure Plot	28	Probability of Survival	31
Distribution Function	25	Sample	18
Endurance Limit Stress Range	13	Serpentine	16
Factorial Testing	33	Standard Deviation	23
Factorial Design	34	Strain (Unit Strain)	6
Failure Level, Failure Rate	30	Stress	1
Fatigue Curve	11	Stress Cycle	5
Fatigue Failure	8	Stress Range	2
Fatigue Life	10	Stress Ratio	14
Fatigue Loading	9	Survival Level, Survival Rate	31
F-distribution	36	Tensile (Young's) Modulus	7
Fractional Factorial Design	35	Variance	22
Loading Pattern	5	Variance Ratio	24
Mean	19	Weibull Plot	25

The following glossary defines briefly various terms from the Mechanics of Materials and statistical fields which may not be familiar to many of the people for whom this report is written. These definitions are slanted to the use in this report and are not complete. For complete definitions and understanding of the terms and the implications in their use, refer to any standard textbook in these fields.

1. Stress - The total load normal to a cross section divided by the cross sectional area. This applies to tensile and compressive stress and the area may be a differential area if the stress varies.

2. Stress Range - The largest algebraic difference between the values of the stress at a point if the stress varies with time.

3. Mean Stress - Here is considered the algebraic average of the largest and smallest values of stress at a point, if the stress varies with time.

4. Bending Stress - The stress induced in the belt as it changes direction. The outside layer is extended and the inside layer is compressed.

5. Stress Cycle - (loading pattern) The repeated pattern of stresses experienced by a point. May be repeated one or more times in one belt revolution.

6. Unit Strain - The change in length per unit length; the total change in length divided by the original length.

7. Tensile Modulus of Elasticity, or Young's Modulus of Elasticity - The ratio of stress to unit strain; also the slope of a stress-unit strain curve.

8. Fatigue Failure - A failure which results from the repeated application of varying stress which is lower than the yield strength of the material.

9. Fatigue Loading - A pattern of stress variation which results in a fatigue failure.

10. Fatigue Life - The number of applications of (cycles of) a fatigue loading cycle which a material undergoes before failure.

11. Fatigue Curve - A plot of the fatigue lives as a function of different levels of a stress pattern. The fatigue life is usually plotted on the abscissa with a logarithmic scale.

12. Bias - Any chosen measure of the stress cycle (loading pattern) used as a reference. Frequently, the mean stress is considered the bias, but results of this report indicate the algebraic minimum stress to be the valid reference.

13. Endurance Limit Stress Range - That stress range which results in an expected fatigue life of 10^7 cycles at a particular bias level. The Endurance Limit Stress Range is considered to be determined by the bias. (The 10^7 cycles value has been determined by experience to be a threshold value, beyond which fatigue failures become insignificant for some materials). Fully reversed or unidirectional stress ranges are easily obtained with existing fatigue testing machines.

14. Stress Ratio - A coined term which has been introduced in this report. The ratio of a stress range with a given bias to the endurance limit stress range with the same bias.

15. Belt Length - Circumference of the belt under a stated load.

16. Serpentine - A belt loading pattern resulting when the belt path includes a reverse bend. (The outside belt layer at one bend becomes the inside layer at another).

17. Population - The infinitely large group of items which can be considered to have some characteristic in common. The identification of the common characteristic is dependent upon the use intended. This might be all males between 18 and 45 years of age, or all 1/16 inch diameter bearing balls, or the grades in Freshman English, etc.

18. Sample - A finite sized group selected at random from the population.

19. Mean - (Average) — The arithmetic average of the values of the variable. The term may apply to the population or to a sample from the population. As the sample size increases the sample mean approaches the population mean as a limit.

20. Median - — The value of the variable which divides all the values of the variable exactly in half. The probability is exactly one half that an individual value of the variable will be higher than or lower than the median. The term may apply to the population or to a sample from the population. As the sample size increases the sample median approaches the population median as a limit.

21. Mode - — The most frequently occurring value of the variable. It is possible for two or more modes to exist, but this is usually the result of mixing that number of populations.

Note: For populations with symmetrical density functions, the mean, median and mode coincide.

22. Variance - — A measure of variability. The arithmetic average of the squares of the deviations of the values of the variable from a particular value of the variable. The variance is normally calculated from the mean of the variable. When the variance is calculated from the mean it has the smallest possible value.

23. Standard Deviation - — A measure of variability. The square root of the variance.

Note: The above definitions are exact for populations. If computed for a sample, the sum of squares is divided by one less than the sample size to obtain a better estimate of the population parameters.

24. Variance Ratio - — A comparison of the variabilities of two or more samples. This is usually to determine whether two or more samples can be considered to come from the same population. Here we use $\frac{\text{larger variance}}{\text{smaller variance}}$

25. Probability Distribution Function - (Weibull plot)

The mathematical description of the probability that a value X or some lesser value will occur in a test. May be shown as a graph, where the ordinate Y is the probability that some value less than or equal to the corresponding abscissa X will result from a particular test.

The difference between any two ordinates is the probability that the test result will lie between the two corresponding abscissa. The distribution function is zero at minus infinity and unity at plus infinity, and changes somewhere in between. In a Normal Distribution the ordinate increases smoothly and continuously from minus infinity to plus infinity with asymptotes at zero and one. In a Weibull distribution the ordinate is zero for all negative values of the variable and rises smoothly and continuously to an asymptote at one, but need not be asymptotic at zero.

26. Confidence Level -

The minimum portion or percentage of the times that the statement made will be correct. This is also the probability that the statement is correct. (Here 95% of the time the belt will live at least as long as predicted).

27. Confidence Band - (Prediction interval)

A range of values which is expected (with a stated probability) to include the test value. (Here there is 90% probability that belt life figures will lie inside the band on each Weibull Plot).

28. Cumulative Failure Plot -

The Weibull Distribution used in failure experiments where life is plotted against relative number of failures.

29. Portion Failed - The relative number of failures (e.g., 1 out of 5). This figure may be modified to satisfy statistical limits.

30. Failure Level - or Failure Rate - The number of failures divided by the sample size.

31. Survival Level - or Survival Rate (Probability of Survival) - The number of items which have not yet failed divided by the sample size. The failure level and the survival level add to one. Both of these terms are applied to a given degree or time of exposure at a given level of stress.

32. Correlation Plot - A graph of 2 values to determine the relationship between them.

33. Factorial Testing - Method of testing a function of several variables for the effect of each, without testing the variables separately.

34. Factorial Design - The actual combinations of conditions in a Factorial Test.

35. Fractional Factorial Design - A more economical but less informative set of combinations than a full Factorial Design. A selected set from a Factorial Design.

36. F-Distribution - A mathematical description of the maximum values expected for a variance ratio if the effects were due to random sampling variation alone. It is a function of sample sizes and desired confidence level in the test.

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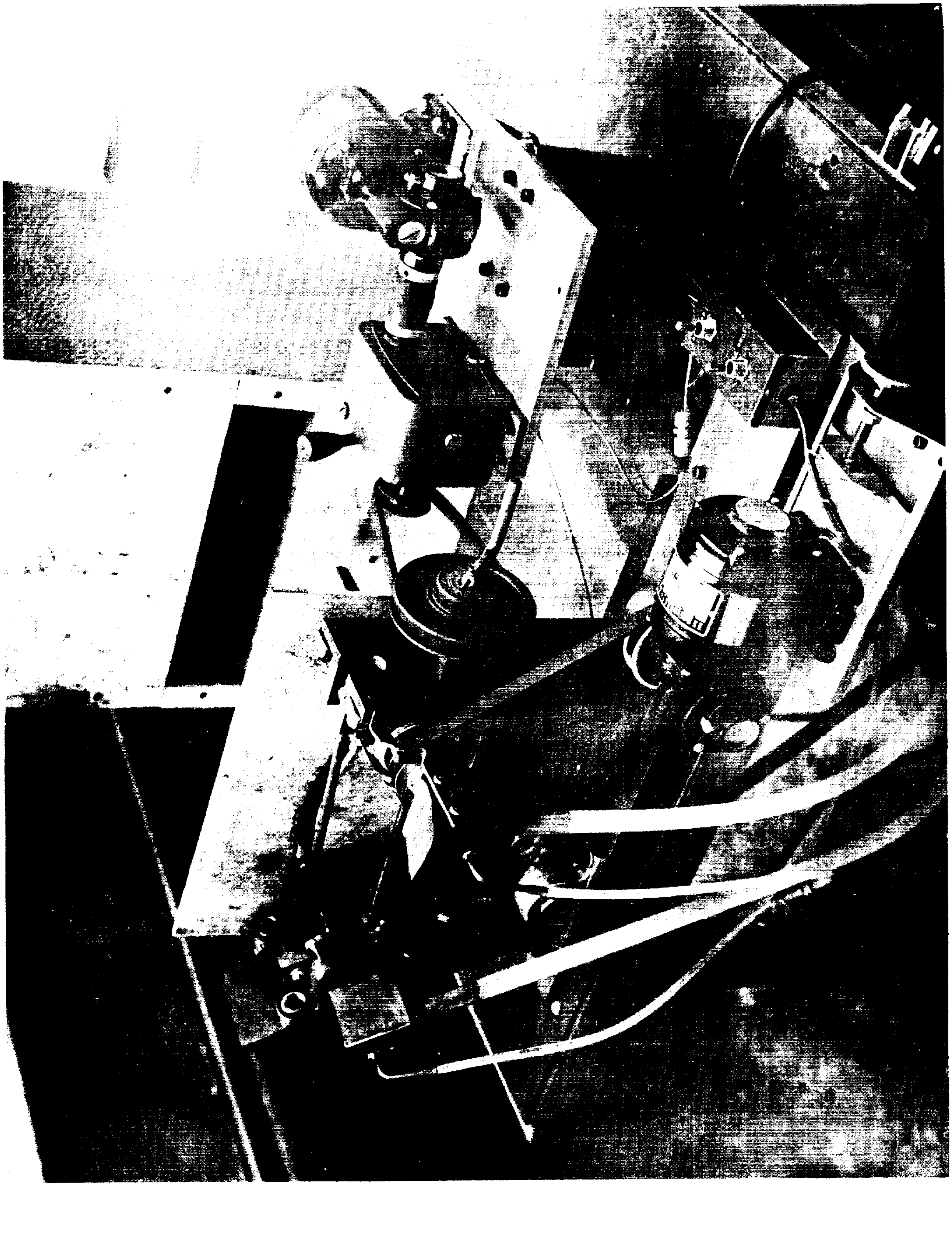
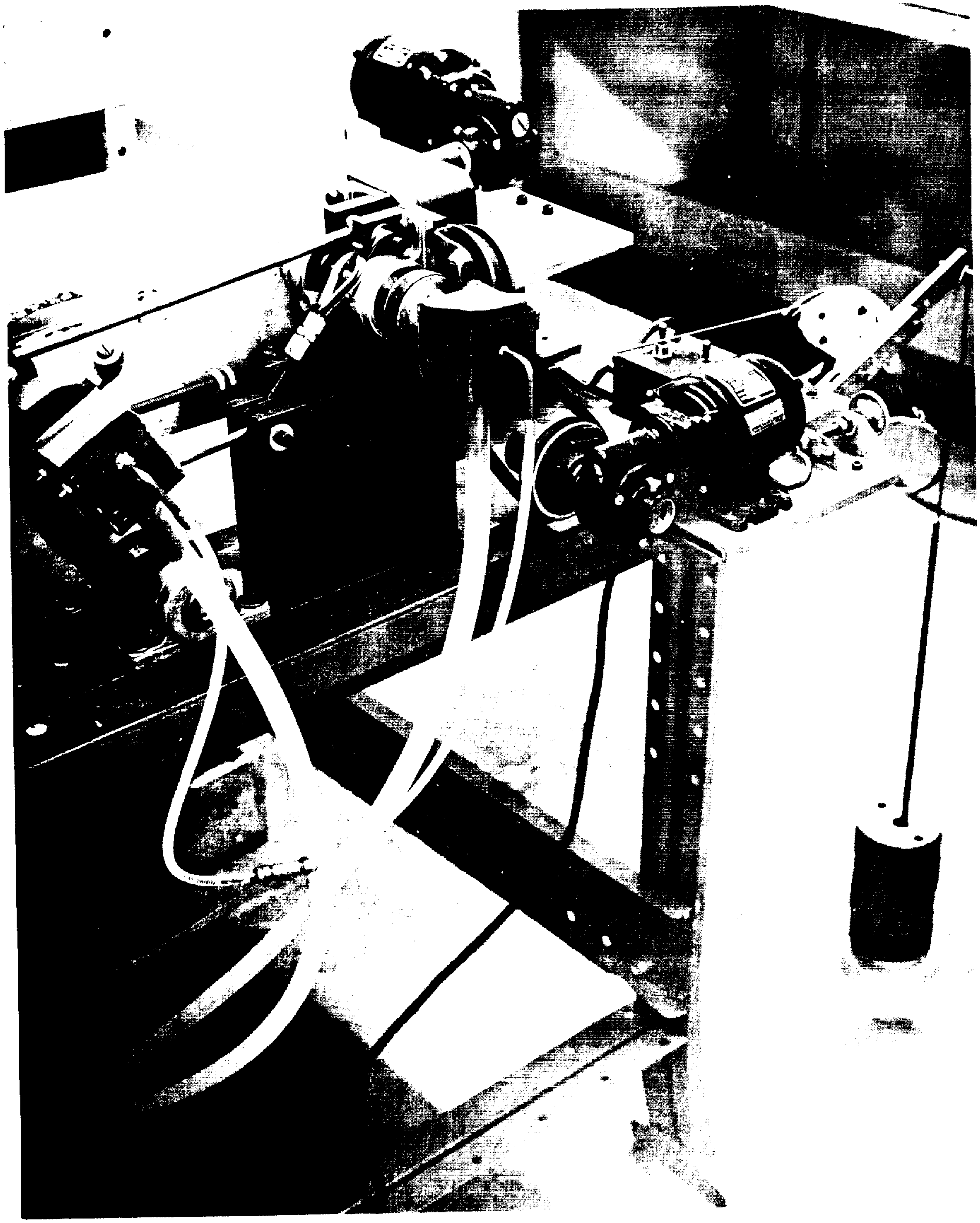
BELT FABRICATING MACHINE
Figure 1.



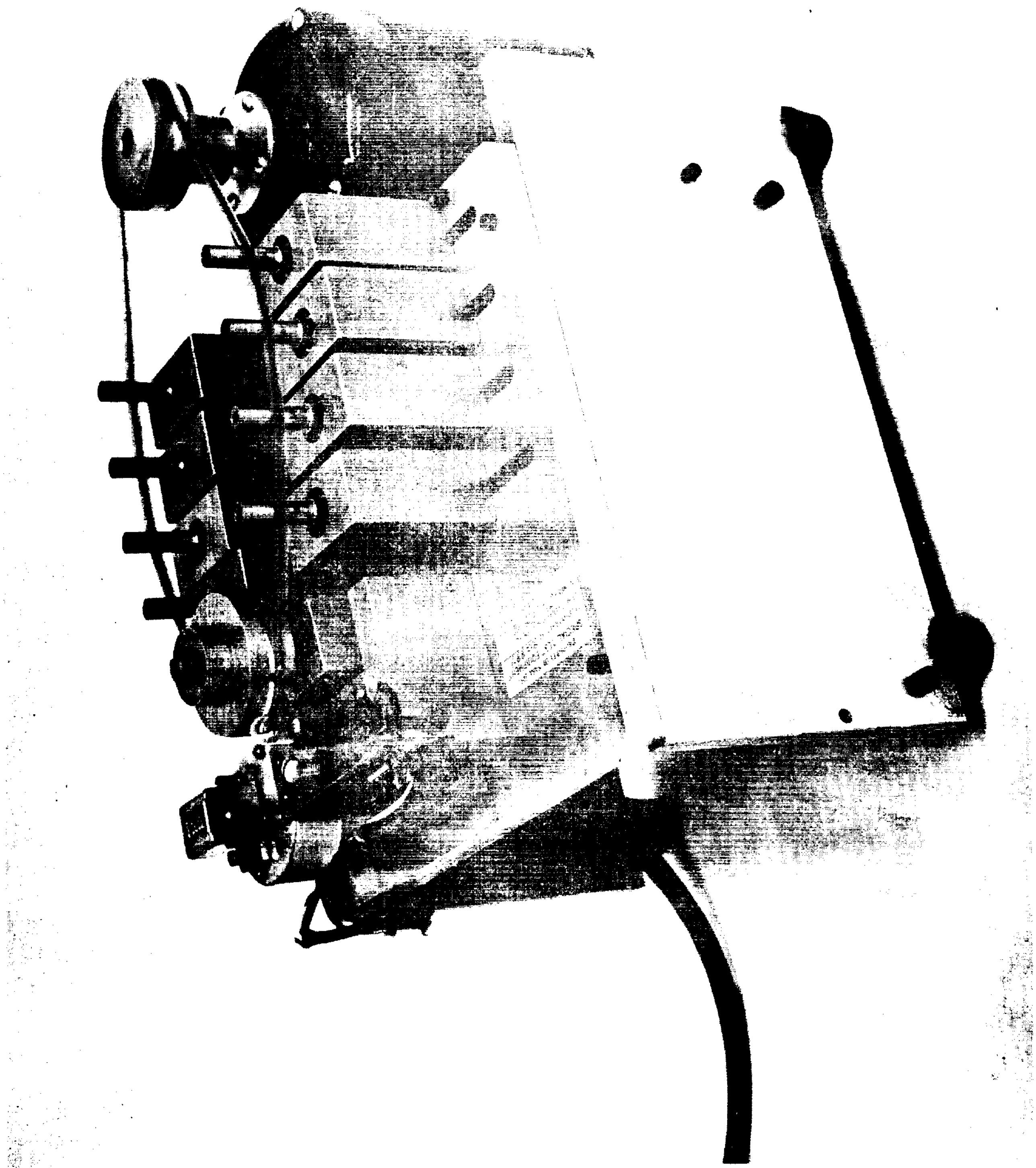
BELT FABRICATING MACHINE

Figure 2.



BELT FABRICATING MACHINE

Figure 3.



BELT TESTER
Figure 4.

FATIGUE LIFE TEST DATA SHEET

Test No. ____________ Belt made date ________________

Test Combination No. ____________ Material ______________________

	DESIGN	FABRICATION
Heat Treat Temperature	__________	__________
Material Thickness	__________	__________
Elongation	__________	__________
L/W Ratio	__________	__________
Stress Method	Constant _____	Constant _____
Heat Treat Time	__________	__________
Belt Width	__________	__________
		TEST
Operating Temperature	__________	__________
Stress Ratio	__________	
Cycle Rate	_______ CPM	
Test Spindle Diameter	__________	__________
Motor Pulley Diameter	_______ nom	_______ actual
Threading Pattern	__________	__________
Belt Length at 1 lb.		__________
Load on Pulley	__________	__________
Belt Angle Loaded		__________

Test Started Date ________________ Time Meter Reading ________________ MIN

Test Ended Date ________________ Time Meter Reading ________________ MIN

Length of Run ________________ MIN

Belt Life ________________ Stress Cycles

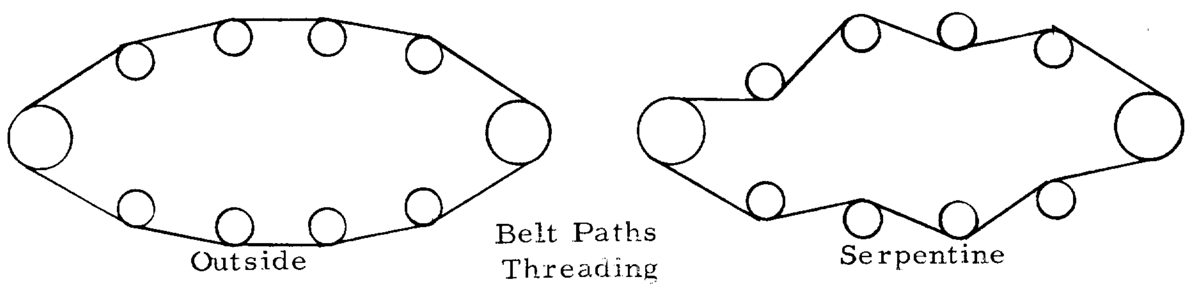
FIGURE 5

APPENDIX A

SUMMARY OF TEST CONDITIONS AND RESULTS

TABLE 1

E - Polyester material. I - Polyimide material.
Thickness, Test Pulley - belt thickness, pulley diameters (inches)
CPM - cycles of stress per minute
* - Test stopped after 10^8 cycles, no failure
All belts were 22.1" long, .187" wide



Belt Paths Threading

Material	E	E	E	E	E	E	E	E
Thickness	.0005	.0005	.0005	.001	.001	.001	.001	.001
Test Pulley	.100	.100	.100	.200	.200	.200	.200	.100
Installed stress	2,250	3,000	4,375	2,250	3,000	4,375	3,700	5,500
Rate CPM	1,940	1,940	1,940	1,940	1,940	1,940	3,880	3,880
Threading	Serp	Serp	Serp	Serp	Serp	Serp	Out	Out
Life in	6.45	6.65	4.91	2.69	1.94	3.49	49.9	56.6
10^5 cycles	20.8	8.02	11.0	5.75	5.58	22.4	218	292
5 tests	32.6	10.7	11.3	18.0	9.00	100	306	420
	51.5	21.2	45.4	126	30.0	103	813	439
	354	82.5	*	217	66.5	128	*	443

Material	E	E	E	E	E	E	E	E
Thickness	.001	.002	.002	.002	.002	.002	.002	.002
Test Pulley	.100	.250	.250	.250	.200	.200	.200	.100
Installed stress	6,875	2,400	3,100	4,000	3,700	5,500	6,875	1,500
Rate CPM	1,940	3,880	3,880	3,880	3,880	3,880	1,940	3,880
Threading	Serp	Out	Out	Out	Out	Out	Serp	Out
Life in	4.45	6.03	159	430	226	36.0	3.48	463
10^5 cycles	6.72	*	282	485	241	44.5	3.56	962
5 tests	7.70	*	478	788	798	97.9	3.88	970
	9.55	*	*	*	903	108	4.72	1,030
	27.6	*	*	*	*	*	18.2	*

TABLE 1 (Continued)

Material	E	E	E	E	E	E	E	E
Thickness	.003	.003	.003	.003	.003	.003	.003	.005
Test Pulley	.375	.375	.375	.300	.300	.200	.200	.500
Installed Stress	2,400	3,100	4,000	3,700	5,500	3,625	4,050	3,700
Rate CPM	3,880	3,880	3,880	3,880	3,880	3,880	1,940	3,880
Threading	Out	Out	Out	Out	Out	Out	Serp	Out
Life in	115	1,010	178	447	44.7	25.5	1.11	37.2
10^5 cycles	724	*	213	702	76.1	55.7	1.49	69.5
5 tests	*	*	361	*	139	77.2	1.77	236
	*	*	382	*	255	99.0	2.09	350
	*	*	*	*	350	749	2.74	*

Material	E	E	E	I	I	I	I	I
Thickness	.005	.005	.005	.001	.001	.001	.001	.010
Test Pulley	.500	.500	.250	.200	.100	.100	.100	1.878
Installed Stress	5,500	7,200	1,500	4,375	3,700	3,050	6,875	3,000
Rate CPM	3,880	3,880	3,880	1,940	3,880	1,940	1,940	984
Threading	Out	Out	Out	Serp	Out	Serp	Serp	Serp
Life in	26.7	32.8	15.9	7.27	264	9.02	5.55	592
10^5 cycles	32.0	60.0	26.1	7.30	338	25.6	9.05	*
5 tests	40.6	63.0	222	130	435	30.6	40.0	*
	42.1	79.6	338	144	480	94.6	46.1	*
	83.7	87.1	416	214	850	95.5	69.0	*

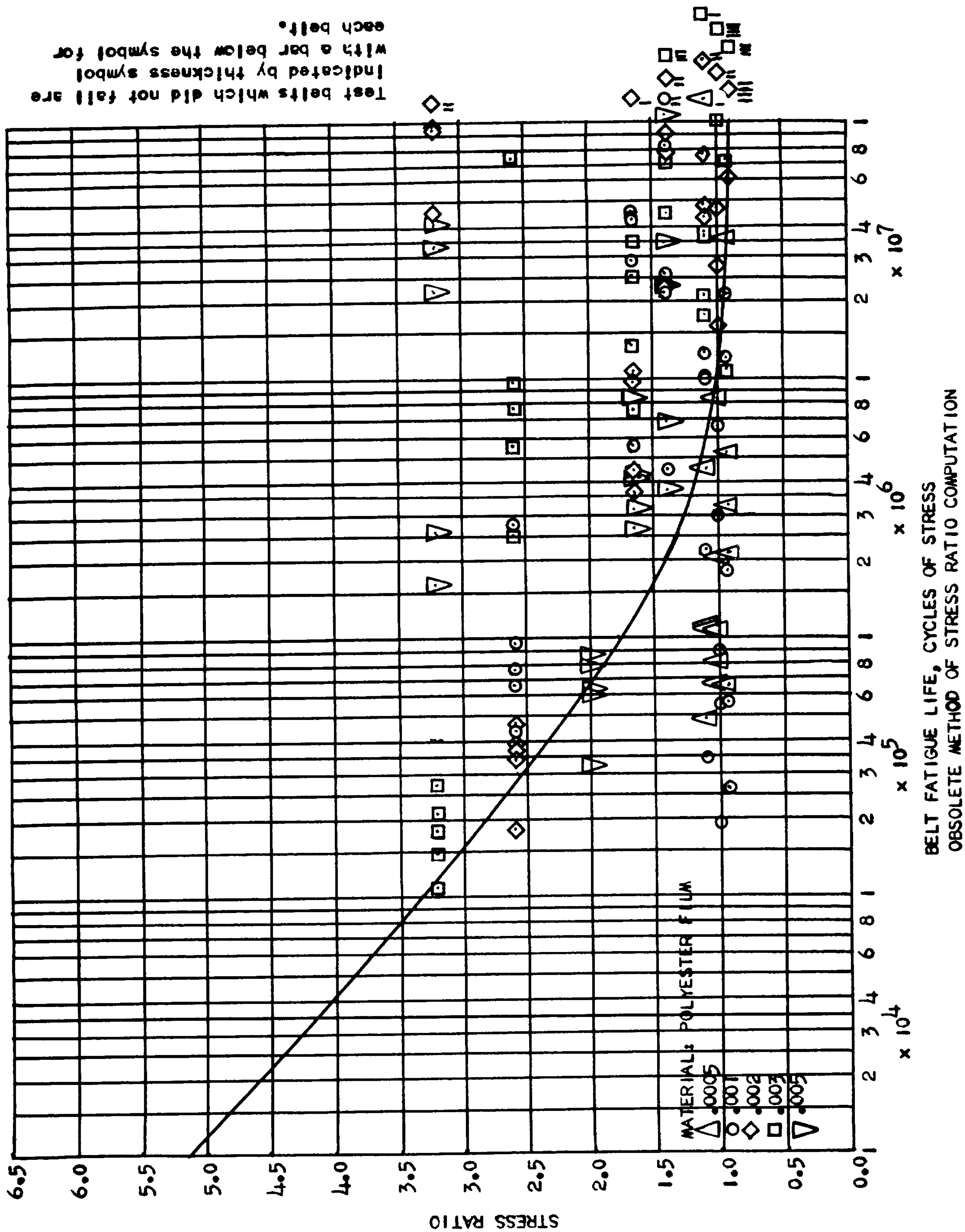

STRESS RATIO
6.5
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
BELT FATIGUE LIFE, CYCLES OF STRESS
OBSOLETE METHOD OF STRESS RATIO COMPUTATION
x 10^4
x 10^5
x 10^6
x 10^7
MATERIAL: POLYESTER FILM
.0005
.001
.002
.003
.005
Test belts which did not fail are
indicated by thickness symbol
with a bar below the symbol for
each belt.

Figure 6

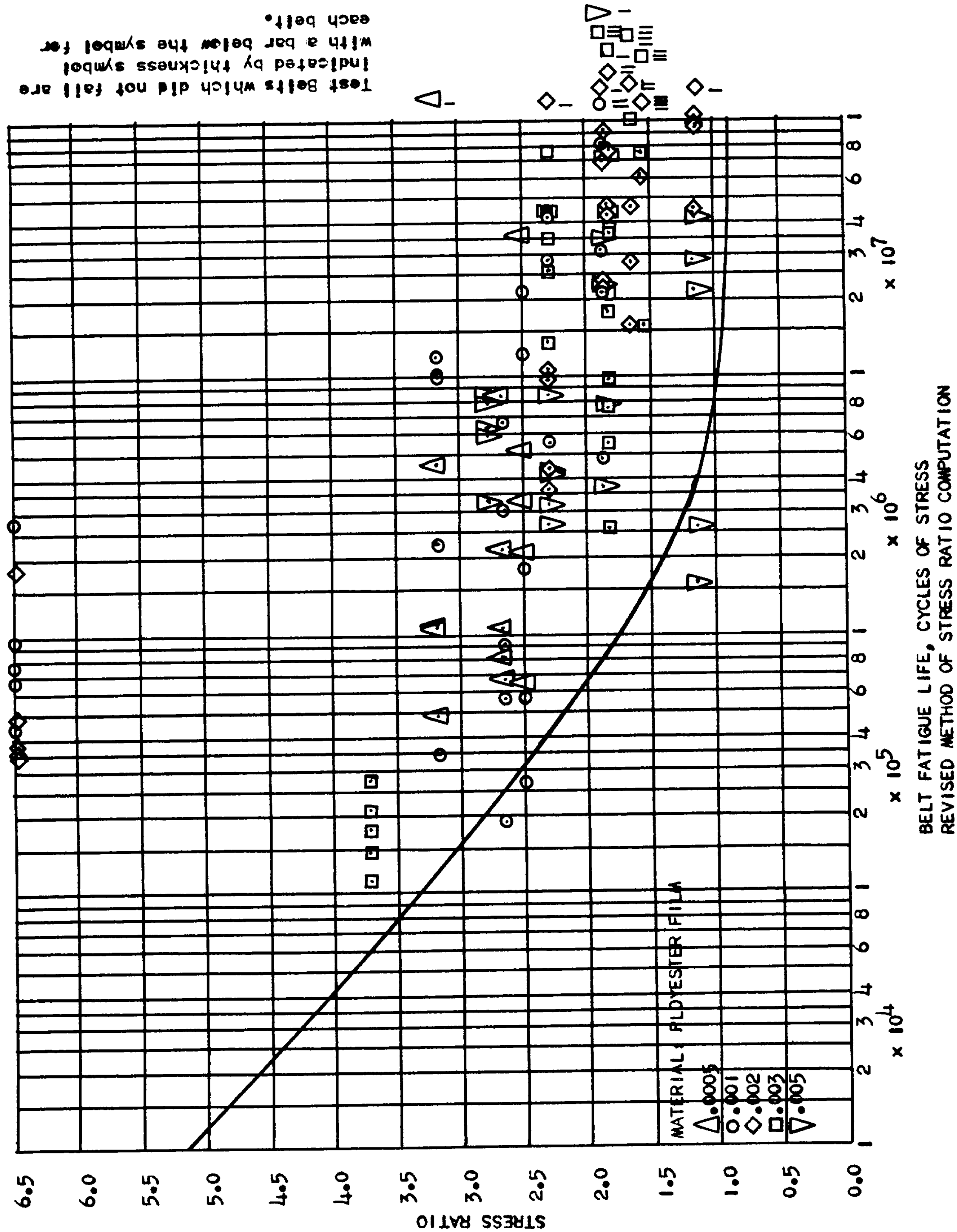

Test Belts which did not fail are indicated by thickness symbol with a bar below the symbol for each belt.
MATERIAL: POLYESTER FILM
.0005
.001
.002
.003
.005
STRESS RATIO
BELT FATIGUE LIFE, CYCLES OF STRESS
REVISED METHOD OF STRESS RATIO COMPUTATION

Figure 7

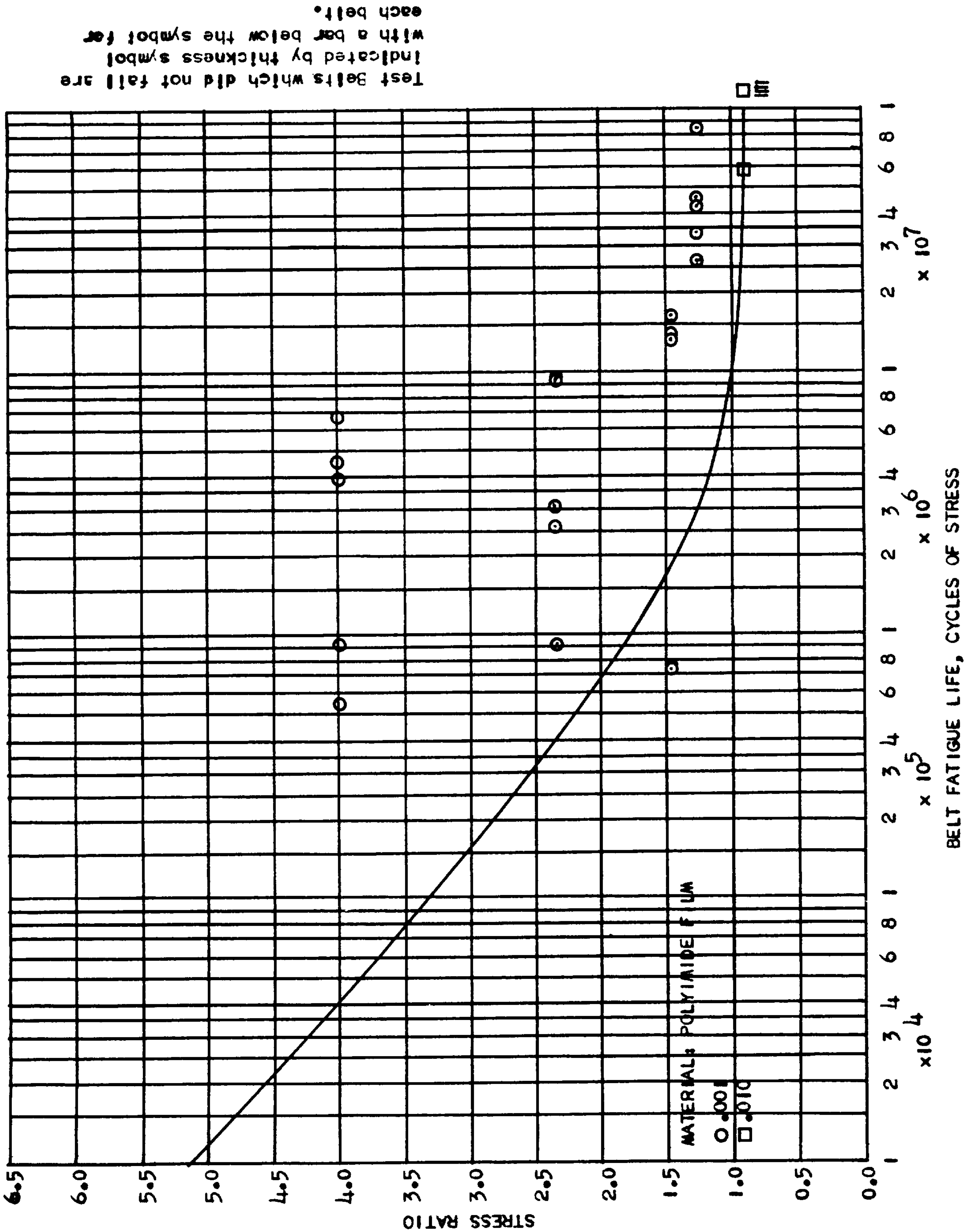

Test Belts which did not fail are
indicated by thickness symbol
with a bar below the symbol for
each belt.
STRESS RATIO
BELT FATIGUE LIFE, CYCLES OF STRESS
MATERIAL: POLYIMIDE FILM
O .001
□ .010

Figure 8

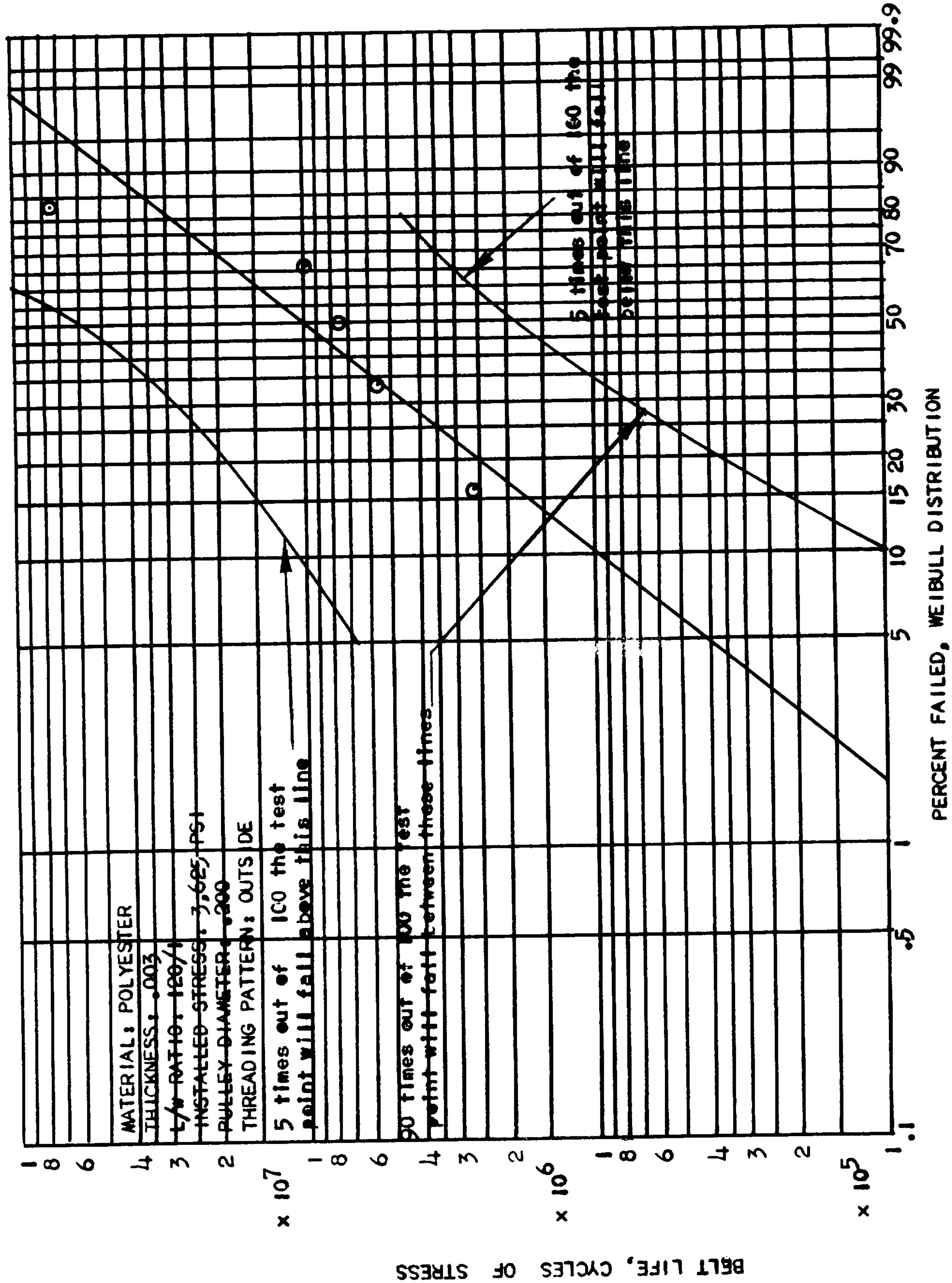

MATERIAL: POLYESTER
THICKNESS: .003
L/W RATIO: 120/1
INSTALLED STRESS: 3,625 PSI
PULLEY DIAMETER: .200
THREADING PATTERN: OUTSIDE
5 times out of 100 the test point will fall above this line
90 times out of 100 the test point will fall between these lines
5 times out of 100 the test point will fall below this line
PERCENT FAILED, WEIBULL DISTRIBUTION
.1
.5
1
5
10
15
20
30
50
70
80
90
99
99.9
BELT LIFE, CYCLES OF STRESS
x 10^5
x 10^6
x 10^7

FIGURE 9

FIGURE 10

MYLAR BELT STUDY, REGRESSION ANALYSIS WORKSHEET FOR SAMPLE OF 5

MAT'L: MYLAR THICKNESS: 3 mil. INSTALLED STRESS 3,625 psi L/W: 120:1 ELONGATION: 11%

	x	y	x^2	y^2	xy
	0.00	0.03	0.00000	0.0009	0.0000
	1.11	1.15	1.2321	1.3225	1.2765
	2.13	1.63	4.5369	2.6569	3.4719
	2.60	1.99	6.7600	3.9691	5.1740
	3.30	4.91	10.8900	24.1081	16.2030
$\sum$	9.14	9.71	23.4190	32.0575	26.1254
Avg	1.8280	1.942	4.68380		

$$\sum (x_i - \bar{x})^2 = \sum x_i^2 - (\sum x_i)^2/n = 23.41900 - 83.5695/5 = 6.71102$$

$$\sum (y_i - \bar{y})^2 = \sum y_i^2 - (\sum y_i)^2/n = \sum y_i^2 - 94.2841/5$$

$$= 32.0575 - 18.85682 = 13.20068$$

$$\sum (x_i - \bar{x})(y_i - \bar{y}) = \sum x_i y_i - (\sum x_i)(\sum y_i)/n = \sum x_i y_i - (\sum y_i) \times 1.8280$$

$$= \sum x_i y_i - 9.71 \times 1.8280$$

$$= 26.1254 - 17.74988 = 8.37552$$

$$\text{Slope} = b = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} = \frac{8.37552}{6.71102} = 1.2480$$

$$\text{Intercept} = a = \bar{y} - b\bar{x} = \bar{y} - 1.2480 \times 1.828$$

$$= 1.942 - 2.281 = 0.339$$

$$\text{Estimate of } y = \tilde{y} = a + bx = 0.339 + 1.2480\ x$$

$$s^2_{y:x} = (1/(n-2))\ (\sum y_i^2 - (\sum y_i)^2/n - b \sum (x_i - \bar{x})(y_i - \bar{y}))$$

$$= (1/3)\ (\_\_\_\_ - \_\_\_\_ - 1.2480 \times 8.37552)$$

$$= (\_\_\_\_ - 13.20068 - 10.452649)/3 = 2.748031/3$$

$$= 0.91601$$

$$s_{y:x} = .9571$$

90% CONFIDENCE BAND ON $y = \tilde{y} \pm tx^o(s_{y:x})$

% failed at x^o	tx^o	$tx^o(s_{y:x})$
80	2.847	2.725
50	2.579	2.468
20	2.920	2.795
10	3.508	3.358
5	4.214	4.033
1	6.054	5.794

95% Confidence Band on slope:

$$= b \pm 1.228\ s_{y/x}$$

$$= b \pm 1.228 \times .9571$$

$$= b \pm 1.175$$

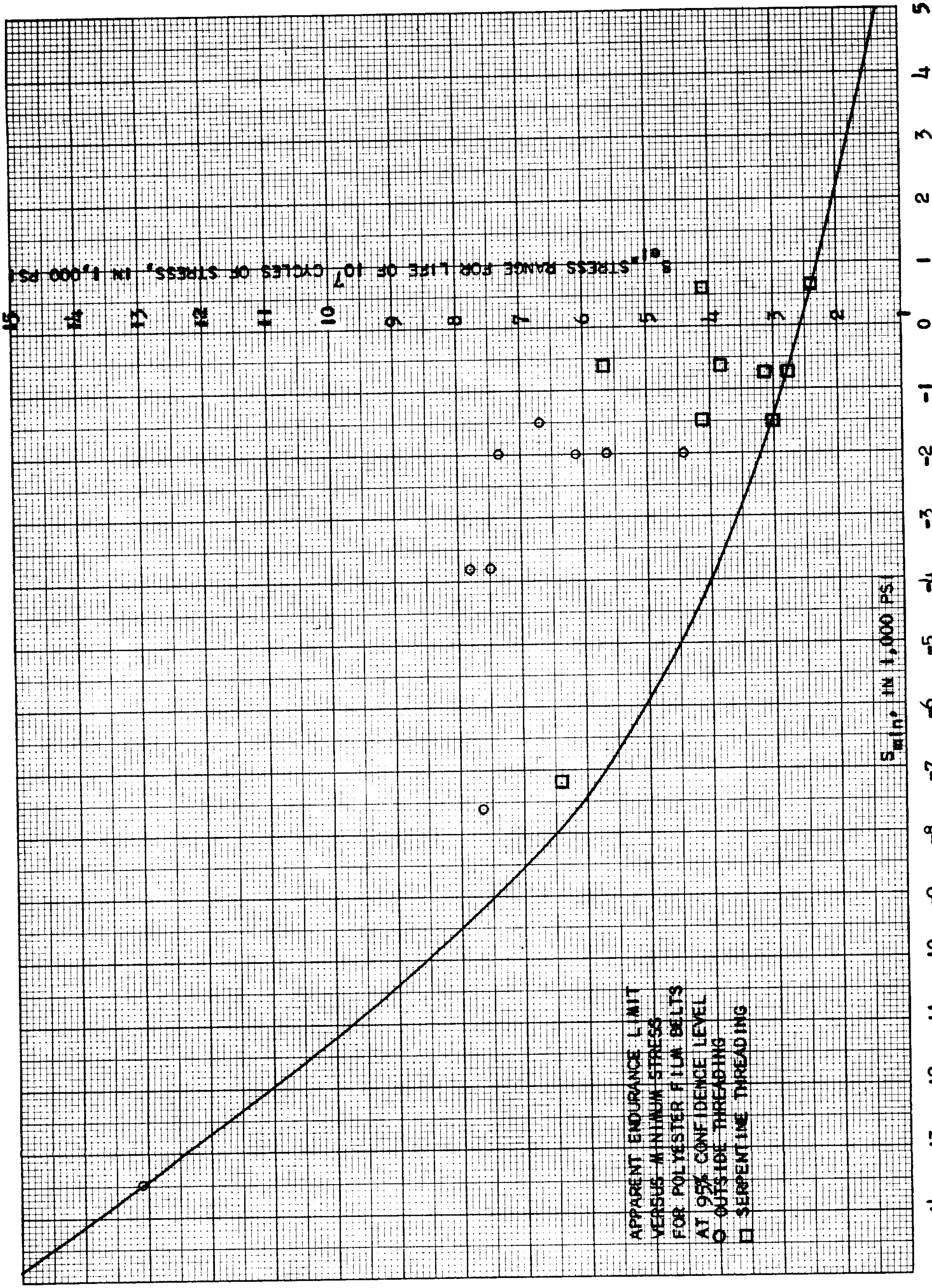

APPARENT ENDURANCE LIMIT
VERSUS MINIMUM STRESS
FOR POLYESTER FILM BELTS
AT 95% CONFIDENCE LEVEL
○ OUTSIDE THREADING
□ SERPENTINE THREADING
S_min, IN 1,000 PSI
S_el = STRESS RANGE FOR LIFE OF 10^7 CYCLES OF STRESS, IN 1,000 PSI
-14 -13 -12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Figure 11

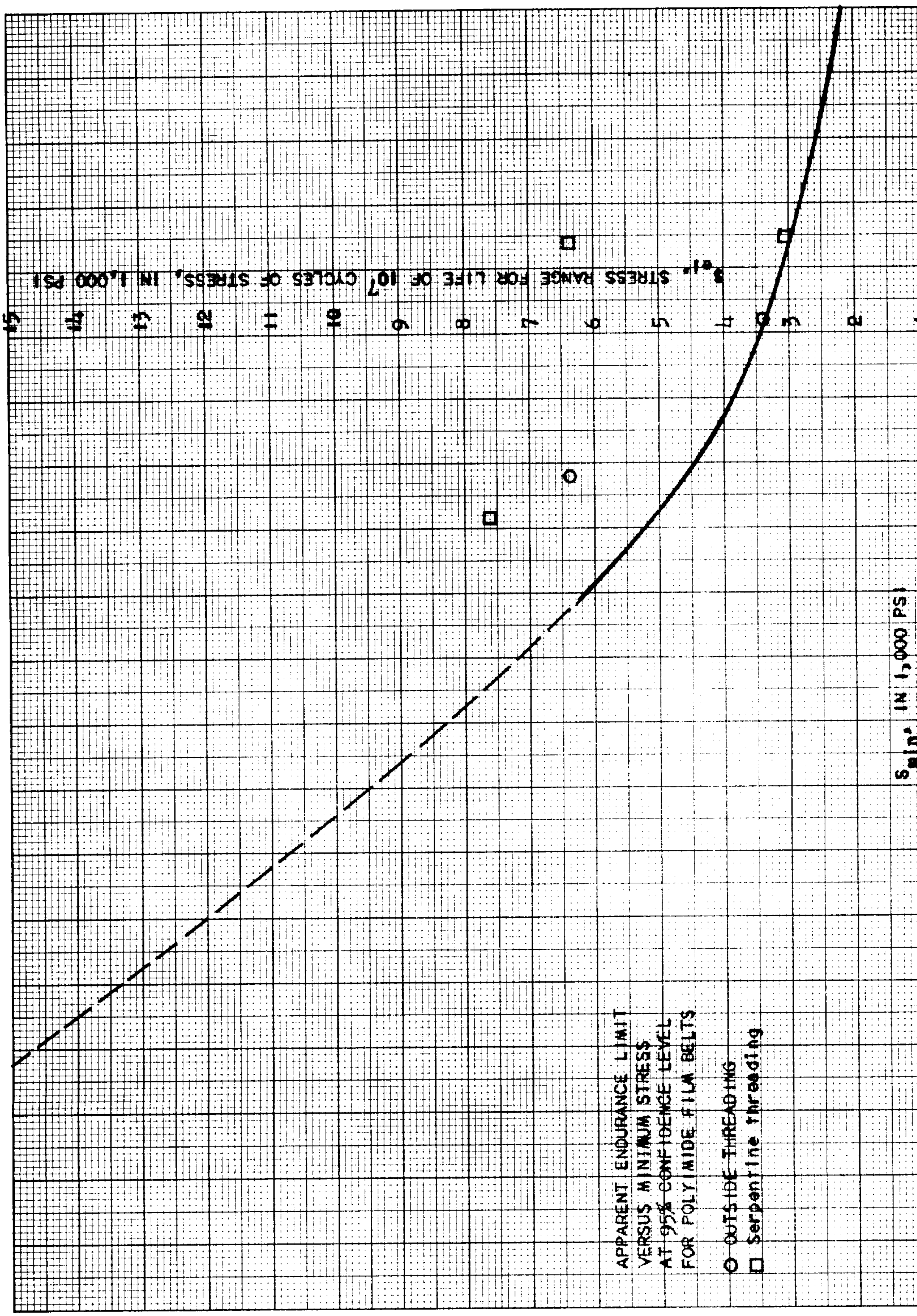

APPARENT ENDURANCE LIMIT
VERSUS MINIMUM STRESS
AT 95% CONFIDENCE LEVEL
FOR POLYIMIDE FILM BELTS
O OUTSIDE THREADING
□ Serpentine threading
S_{el}, STRESS RANGE FOR LIFE OF 10^7 CYCLES OF STRESS, IN 1,000 PSI
S_{min}, IN 1,000 PSI
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
-14
-13
-12
-11
-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5

Figure 12

APPENDIX B

TABLE 2

SUMMARY OF TEST CONDITIONS AND RESULTS FOR FRACTIONAL FACTORIAL TEST

Heat treat temperature	a	275 F.	A	325 F.	
Material thickness	b	.001"	B	.005"	
Elongation	c	35%	C	90%	All belts
Length to width ratio	d	70:1	D	43:1	were
Heat treat time	e	1/2 hour	E	2 hours	22.1" long
Operating temperature	f	Room	F	200 F.	
Stress ratio	g	1.88	G	2.50	
Cycle Rate CPM	h	1,270	H	3,810	

Test Conditions	Life in 10^5 Cycles of stress
AbcDEfgh	241
abcdEFgh	38.2
AbCdefgh	413
abCDeFgh	19.4
aBcDefgh	6.38
ABcdeFgh	192
aBCdEfgh	18.0
ABCDEFgh	13.9
abcdefGh	847
AbcDeFGh	12.2
abCDEfGh	16.5
AbCdEFGh	163
ABcdEfGh	77.5
aBcDEFGh	18.2
ABCDefGh	7.57
aBCdeFGh	160
abcdefgH	443
AbcDeFgH	229
abCDEfgH	350
AbCdEFgH	50.7
ABcdEfgH	90.5
aBcDEFgH	22.4
ABCDefgH	18.2
aBCdeFgH	219
AbcDEfGH	90.2
abcdEFGH	123
AbCdefGH	46.6
abCDeFGH	6.91
aBcDefGH	25.8
ABcdeFGH	10.6
aBCdEfGH	7.95
ABCDEFGH	68.0

TABLE 3

SUMMARY OF ANALYSIS OF VARIANCE OF FRACTIONAL FACTORIAL TEST

Variable	Variance Ratio	D.F.	F-Value 90%	95%	99% Sig.
A. Heat Treat Temperature	0.2053	1/30	2.88	4.17	7.56
B. Material Thickness	5.0391*				
C. Elongation	0.7964				
D. L/W Ratio	5.6508*				
E. Heat Treat Time	0.0107				
F. Operating Temperature	0.2379				
G. Stress Ratio	1.4287				
H. Cycling Rate	0.0589				
Interaction					
AB	1.6553	3/28	2.29	2.95	4.57
AC	0.3156				
AD	2.1315				
AE	0.5283				
AF	0.1400				
AG	0.6510				
AH	0.2205				
BC	2.1934				
BD	4.0453*				
BE	1.5986				
BF	4.3523*				
BG	2.3551+				
BH	1.5934				
CD	2.1101				
CE	0.2524				
CF	0.8132				
CG	0.7302				
CH	0.2695				
DE	4.4980*				
DF	1.8905				
DG	2.4469+				
DH	3.3000*				
EF	0.0779				
EG	0.7011				
EH	0.1163				
FG	0.5605				
FH	0.0929				
GH	1.1327				

*Clearly significant
+ Probably significant

TABLE 4

SUMMARY OF SAMPLE MEANS AND PERCENTAGE CHANGES OF SIGNIFICANT VARIABLE AND TWO FACTOR INTERACTIONS

For single variable each mean has a sample size of 16

For two factor Interactions, each mean has a sample size of 8

Single Variable

Material thickness	.001"	.005"
Cycles of stress	10.38×10^6	6.57×10^6
Percent	100%	-37%
Length to width ratio:	70:1	40:1
Cycles of stress	10.5×10^6	6.49×10^6
Percent	100%	-38%
Stress Ratio:	1.88	2.50
Cycles of Stress	9.39×10^6	7.26×10^6
Percent	100%	-22 1/2%

TABLE 4 (continued)

Two Factor Interactions

L/W	Thickness .001"	Thickness .005"
70:1	13.14×10^5 100%	8.39×10^5 -36%
40:1	8.19×10^5 -37 1/2%	5.14×10^5 -61%
Operating Temp:	Thickness .001"	Thickness .005"
Room	14.10×10^5 100%	5.42×10^5 -61%
200 F.	7.69×10^5 -45%	7.96×10^5 -43%
Stress Ratio:	Thickness .001"	Thickness .005"
1.88	12.6×10^5 100%	6.99×10^5 -44 1/2%
2.5	8.54×10^5 -32%	6.18×10^5 -51%
Heat Treat Time	L/W --70:1	L/W--40:1
1/2	13.6×10^5 100%	5.14×10^5 -62%
2	8.12×10^5 -40%	8.21×10^5 -39 1/2%
Stress Ratio	L/W--70:1	L/W--40:1
1.88	11.5×10^5 100%	7.66×10^5 -33 1/2%
2.5	9.58×10^5 -17%	5.50×10^5 -52%
Cycling rate	l/W--70:1	L/W--40:1
1,270	12.4×10^5 100%	5.22×10^5 -54%
3,810	8.90×10^5 -22 1/2%	8.08×10^5 -30%

APPENDIX C

Procedure for Calculating Fatigue Life of Drive Belts

The procedure to be followed for a simple two pulley system will be given and illustrated first. The added considerations for multiple pulley and serpentine belt arrangements will then be discussed and illustrated.

1. Enter the application of the given design in the box on the front of the work sheet, Figure 13 (Pg. 74)

2. On the back of the work sheet, Figure 14 (Pg. 75) calculate the changes in stress due to the torque transmitted and the bending at the smallest pulley.

3. To determine the maximum and minimum values of the stress on the inner side (the side in contact with the pulleys) these stresses are combined with the installed stress in the appropriate blocks on Fig. 14 (Outside pattern Max Stress; Minimum Stress). A tensile stress is positive, - compressive stress negative, and the algebraic values are used. The maximum and minimum stresses are transferred to the front of the worksheet.

4. On the front of the work sheet, calculate the number of belt revolutions per hour.

5. Calculate the stress range (the algebraic difference between the maximum and minimum stress).

6. Refer to Figure 15 or 16 (Pg. 76, 77), the curves of the endurance limit stress range and read the endurance limit stress range for the minimum stress (S_{min}) in the stress cycle. Be careful to use Figure 15 for polyester material, and Figure 16 for polyimide; observe carefully the sign of S_{min} when entering either.

7. The stress range, found in (5) above, is divided by this value of the endurance limit stress range to determine the stress ratio.

8. The fatigue life is then found by entering the fatigue life curve, Figure 17, with the stress ratio and reading the life from the appropriate curve.

   Note: The choice of which curve to use is determined by the reliability desired. 95%, 1 of 2 will provide 50% reliability with 95% confidence (95 times out of 100 neither of 2 belts will fail until after this many cycles). 1 of 5 will provide 80% reliability at 95% confidence, etc. Most probable 1 of 2 is 50% reliability at 50% confidence. This latter is equivalent to the MTTF.

Once the life in cycles is determined it remains to discover the number of cycles in one belt revolution, in order to convert from stress cycles life to hours.

9. Choose any point on the inside of the belt, trace the changes in stress experienced by this point as it moves through the belt path back to the starting point.

10. How many times did the point experience approximately (80% is a good dividing line) the full stress range, S_{max} to S_{min} ? Minor excursions of the stress compared to the calculated stress range do

not count. This whole number of times is the number of stress cycles per revolution.

11. Divide life in stress cycles by stress cycles per belt revolution to get life in belt revolutions, omit this step if there is one stress cycle per belt revolution.

12. Divide belt life in revolutions by belt revolutions per hour to get life in hours.

In the two pulley, speed changing case (one large, one small pulley) the belt endures one stress cycle during each revolution of the belt so the belt life in revolutions is equal to the number of stress cycles. This value is divided by the belt revolutions per hour to get the prediction of life in hours. The proper interpretation of this value is that at least 95% of the time all of the lot of the size specified on the curve will survive at least as long as indicated. The median time to failure is obtained by using the dashed curve which is labeled 'most probable value'. Figures 20 and 21 show a worked-out sample of this type.

In the example covered above the bending stress at the second pulley was appreciably less than that at the driver and was not counted. If the second pulley were equal in size or only slightly larger than the driver, the belt would go through two complete stress cycles before returning to the starting point. With three equal sized pulleys it would be three stress cycles per revolution, etc. In any case, the life in stress cycles is divided by the number of stress cycles per belt revolution to get the life in belt revolutions. A good dividing point is 80%; if the second stress range exceeds 80% of the greatest stress range, count both. All the above applies when the belt bends in one direction only.

In the serpentine arrangement where the belt undergoes a bending reversal it is necessary to determine which side of the belt will have a shorter life. In this arrangement the stress range will be the same on both belt sides; however, the S_{max} and S_{min} will in general be different on each side. Since the Endurance Limit Stress Range decreases as S_{min} increases, the side with the larger algebraic (-1000 psi is larger than -2000 psi) value of S_{min} will determine the shortest lived side of the belt (and therefore the life of the belt itself). To discover which side has the larger S_{min}, consider that S_{min} is produced by compression at a pulley. Each side of the belt is in compression contact with a group of pulleys. Of these two groups, the group of pulleys with the smallest diameter will produce the smallest S_{min}. Use the other side of the belt for life calculations. If there is any doubt, make separate calculations for each side and choose the worst.

For the serpentine arrangement, enter the belt data in Figure 13 and again calculate the stresses at the top of Figure 14. Now use these stresses to calculate S_{max} and S_{min} using the serpentine block for S_{max}.

To find the life in cycles transfer S_{min} and S_{max} to Figure 13 and proceed as for outside pattern belts.

To figure the number of stress cycles per revolution perform the same point trace as for the outside pattern belt. The number of times the point experiences the full range of stresses as the point goes through the belt path back to the starting point, is the number of cycles per revolution. A double maximum or double minimum with no intervening opposite extreme value is not counted as a cycle for the serpentine arrangement. Figures 22 and 23 show a worked out example for a serpentine belt.

In general, after the stress ratio has been calculated, the appropriate value of assured life can be read on Figure 17. The dashed curve gives the most probable value that the median life will exceed. The three solid curves give lives that all of samples of the indicated sizes will exceed at least 95% of the time. This is presented in the usual form of fatigue curves. The specification of reliability requires data which is shown implicitly in the set of curves in Fig. 17. This data has been extracted and shown explicitly in Figures 18 and 19.

Median Time to Failure may be specified as some value which must be exceeded. The procedure above predicts the most probable median life (from the dashed curve). This is equivalent to the mean life in symmetrical distributions. When the predicted median life (taken from the dashed curve in Fig. 17) exceeds the required life the design is considered adequate.

The life requirement may also be specified by stating a required probability of survival ("reliability") for some fixed length of time (which is determined by the mission). The probability of survival of a given design is then determined and it must equal or exceed the required value to be sure of adequate life. This is done by the use of Figs. 18 and 19.

Figure 18 shows the entire life history for various stress ratios. These curves can be read directly for various levels of probability of survival except that the curves interfere with each other at levels above 95%. The life (in cycles of stress) at the required probability of survival (or the probability of survival at the required life) may be read (as desired) and compared to the required value of life (or probability of survival).

Figure 19 expands the upper 10% of the probability of survival scale and normalizes the life scale so a single curve is shown. The median life for the design (the dashed curve in Fig. 17) is divided into the required life to obtain the life ratio and the probability of survival is read. Alternatively, the life ratio can be read for the required probability of survival; the life ratio is then multiplied by the median life to obtain the most probable life at the specified probability of survival.

It should be noted that the ordinate of Figure 19 is an inverted log scale and the numbering should be read carefully to avoid errors. Figures 18 and 19 are obtained by replotting points from the best fit line on a Weibull Plot for each stress ratio for Figure 18. These two Figures, 18 and 19, are therefore at the 50% confidence level.

Date ____________

FATIGUE LIFE CALCULATION WORKSHEET

Name ______________________ Part No. ______________

Belt Length ________ in.	Thickness (t) ________ in.	Width (w) ________ in.
Installed stress S_o _____ psi	Torque: M = ______ lb. in. ________ oz. in. (Convert torque to lb. in. if given in oz. in.)	
Driver Diam. D_d ______ in.	Speed ________ RPM	Required Life: ________ hrs.
Smallest Pulley Diam. D_s ________ in. This data from design or application	Pulley Diam. D ________ in. (Life Controlling)	

Stress Cycles per Belt Rev. = ________

Belt Revolutions per Hour $= \dfrac{3.14 \times D_d \times RPM \times 60}{L}$

$= \dfrac{188\ (\quad)\ (\quad)}{(\quad)} \times$ ________ = ______ Rev/hr.

Stress Range = $S_{max} - S_{min}$ = ( ) - ( ) = ________ psi

Minimum Stress S_{min} ________ psi

Endurance Limit = S_{el} ________ psi from Fig. 15 or 16

Stress Ratio = $\dfrac{\text{Stress Range}}{S_{el}} = \dfrac{(\quad)}{(\quad)}$ = ____________

Life = ____________ cycles of stress from Fig. 17

Belt Life = ____________ revolutions of belt

= ____________ hours (Most Probable Value
(For lot of ________ at 95% confidence level.

______ % Probability of Survival for at least ________ hours of operation

Calculate S_{max} and S_{min} on reverse side

Figure 13

Change in stress due to transmitted torque:

$$S_T = \frac{M}{D_d \ tw} = \frac{(\qquad\qquad)}{(\qquad)(\qquad)(\qquad)} = \_\_\_\_\_\_\_\_\ \text{psi}$$

$E = 7.5 \times 10^5$ for Polyester)

) USE NO OTHER VALUES

$= 5.9 \times 10^5$ for Polyimide)

Bending stress at Smallest Pulley on Life controlling side:

$$S_D = \frac{E \quad t}{D} = \frac{(\quad \times 10^5)(\qquad)}{(\qquad)} = \_\_\_\_\_\_\_\_\ \text{psi}$$

Bending stress at Smallest Pulley in Serpentine System (on opposite side of belt):

$$S_s = \frac{E \quad t}{D_s} = \frac{(\quad \times 10^5)(\qquad)}{(\qquad)} = \_\_\_\_\_\_\_\_\ \text{psi}$$

Max Stress Outside Pattern:

S_o = ________

Plus S_T = ________

S_{max} = ________

Minimum Stress All Patterns:

S_o = ________

Minus S_T = ________

Sub-total = ________

Minus S_D = ________

S_{min} = ________

Max Stress Serpentine Pattern:*

S_o = ________

Plus S_T = ________

Sub-total = ________

Plus S_s = ________

S_{max} = ________

* Note: Base Calculations on side of belt which is in contact with pulleys. If both sides contact pulleys (with reversed bending) use the side of the belt which results in the algebraic highest value of S_{min}.

Figure 14

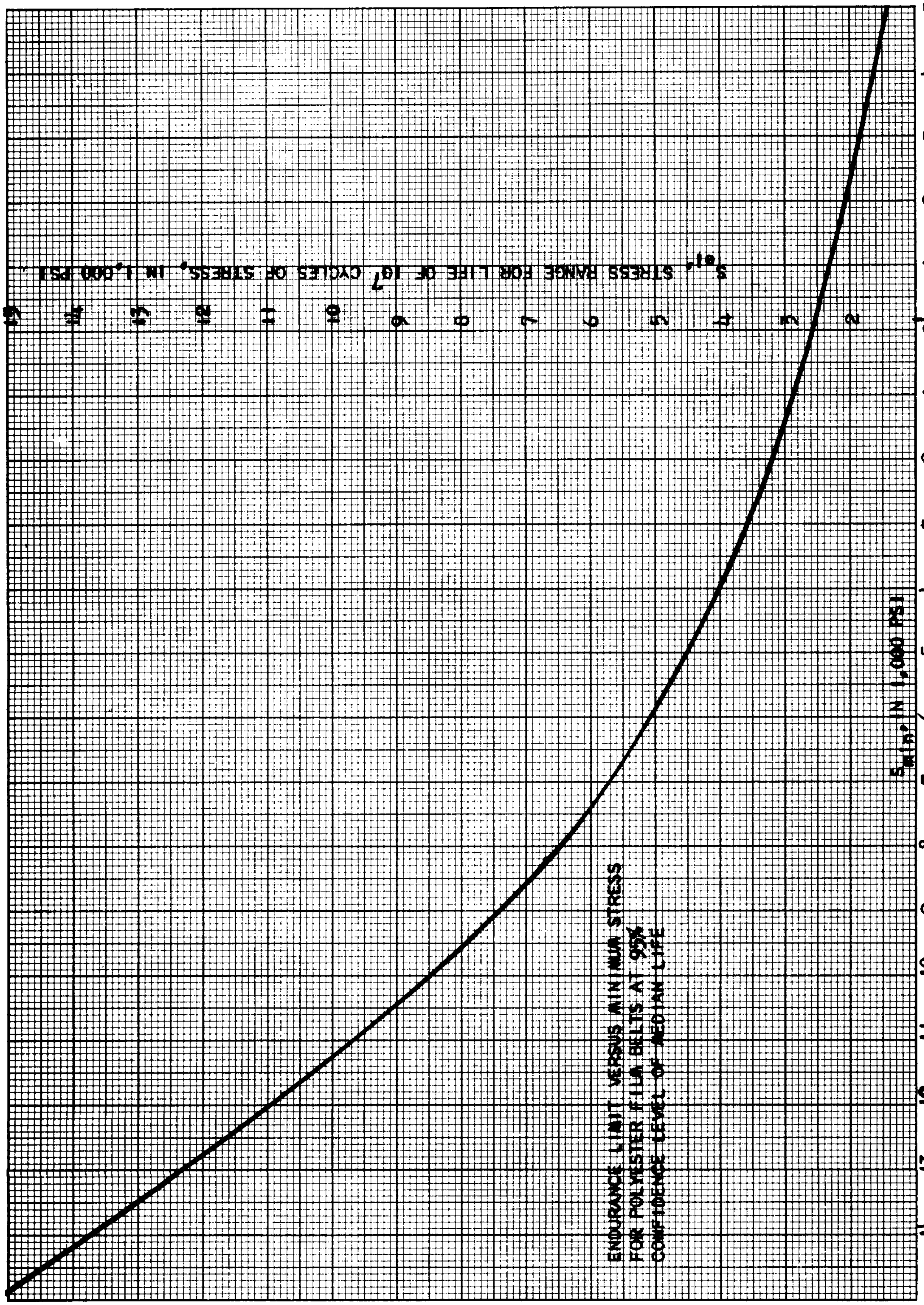

ENDURANCE LIMIT VERSUS MINIMUM STRESS
FOR POLYESTER FILM BELTS AT 95%
CONFIDENCE LEVEL OF MEDIAN LIFE
S_{min}, IN 1,000 PSI
S_{el}, STRESS RANGE FOR LIFE OF 10^7 CYCLES OF STRESS, IN 1,000 PSI

Fig. 15

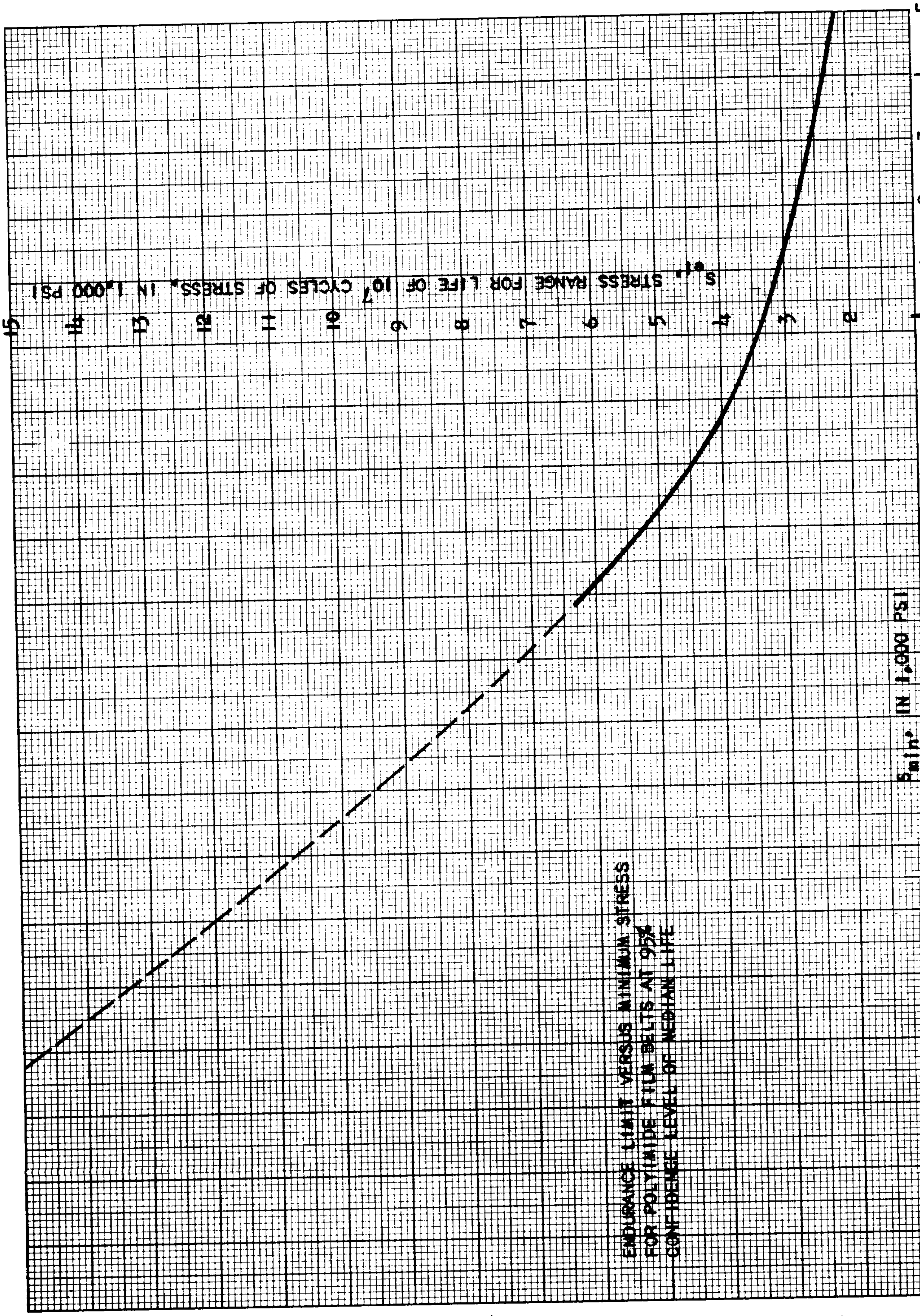

S_e', STRESS RANGE FOR LIFE OF 10^7 CYCLES OF STRESS, IN 1,000 PSI
S_{min}, IN 1,000 PSI
ENDURANCE LIMIT VERSUS MINIMUM STRESS
FOR POLYIMIDE FILM BELTS AT 95%
CONFIDENCE LEVEL OF MEDIAN LIFE

Fig. 16

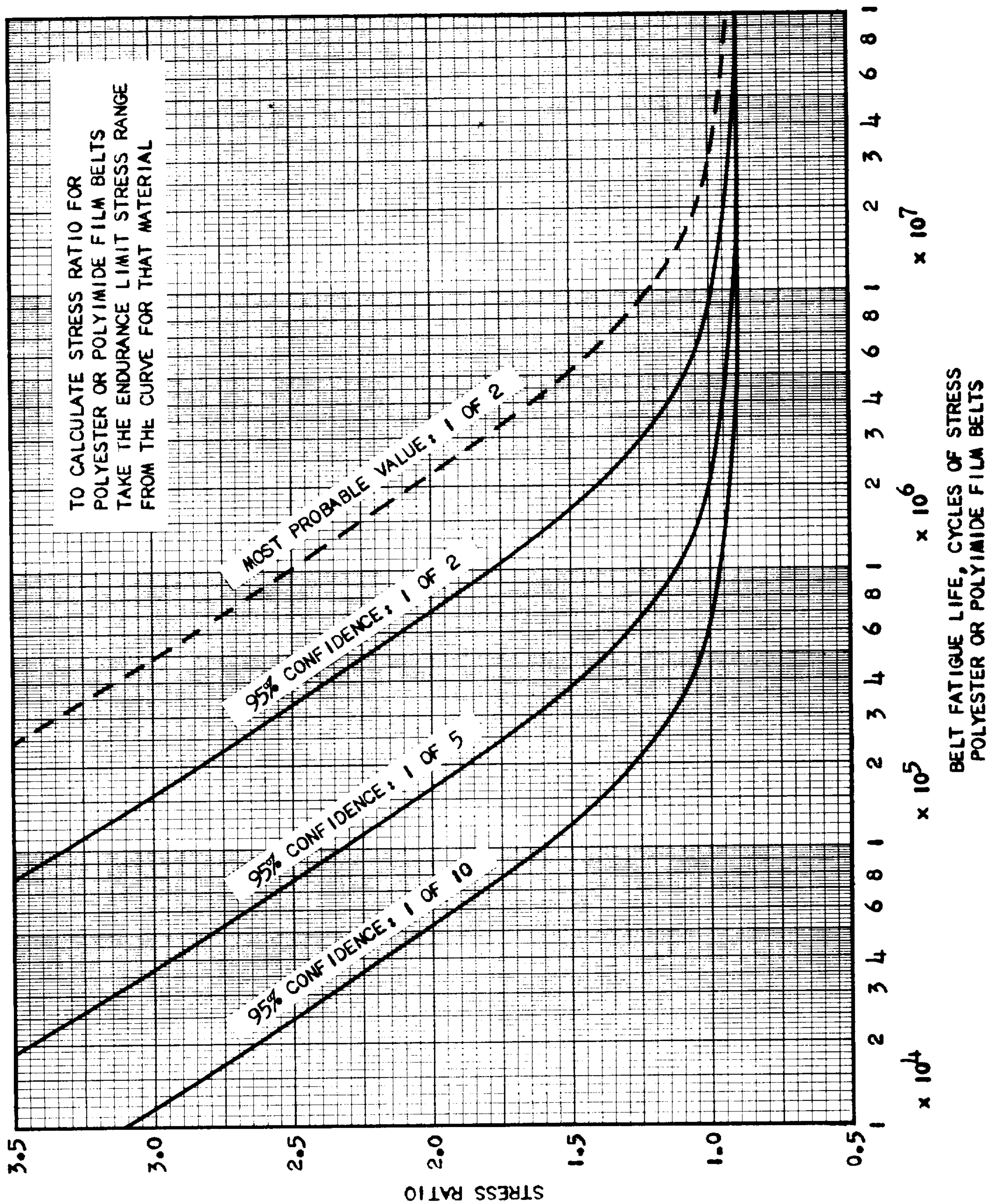

TO CALCULATE STRESS RATIO FOR
POLYESTER OR POLYIMIDE FILM BELTS
TAKE THE ENDURANCE LIMIT STRESS RANGE
FROM THE CURVE FOR THAT MATERIAL
MOST PROBABLE VALUE: 1 OF 2
95% CONFIDENCE: 1 OF 2
95% CONFIDENCE: 1 OF 5
95% CONFIDENCE: 1 OF 10
STRESS RATIO
3.5
3.0
2.5
2.0
1.5
1.0
0.5
BELT FATIGUE LIFE, CYCLES OF STRESS
POLYESTER OR POLYIMIDE FILM BELTS
x 10^4
x 10^5
x 10^6
x 10^7

FIGURE 17

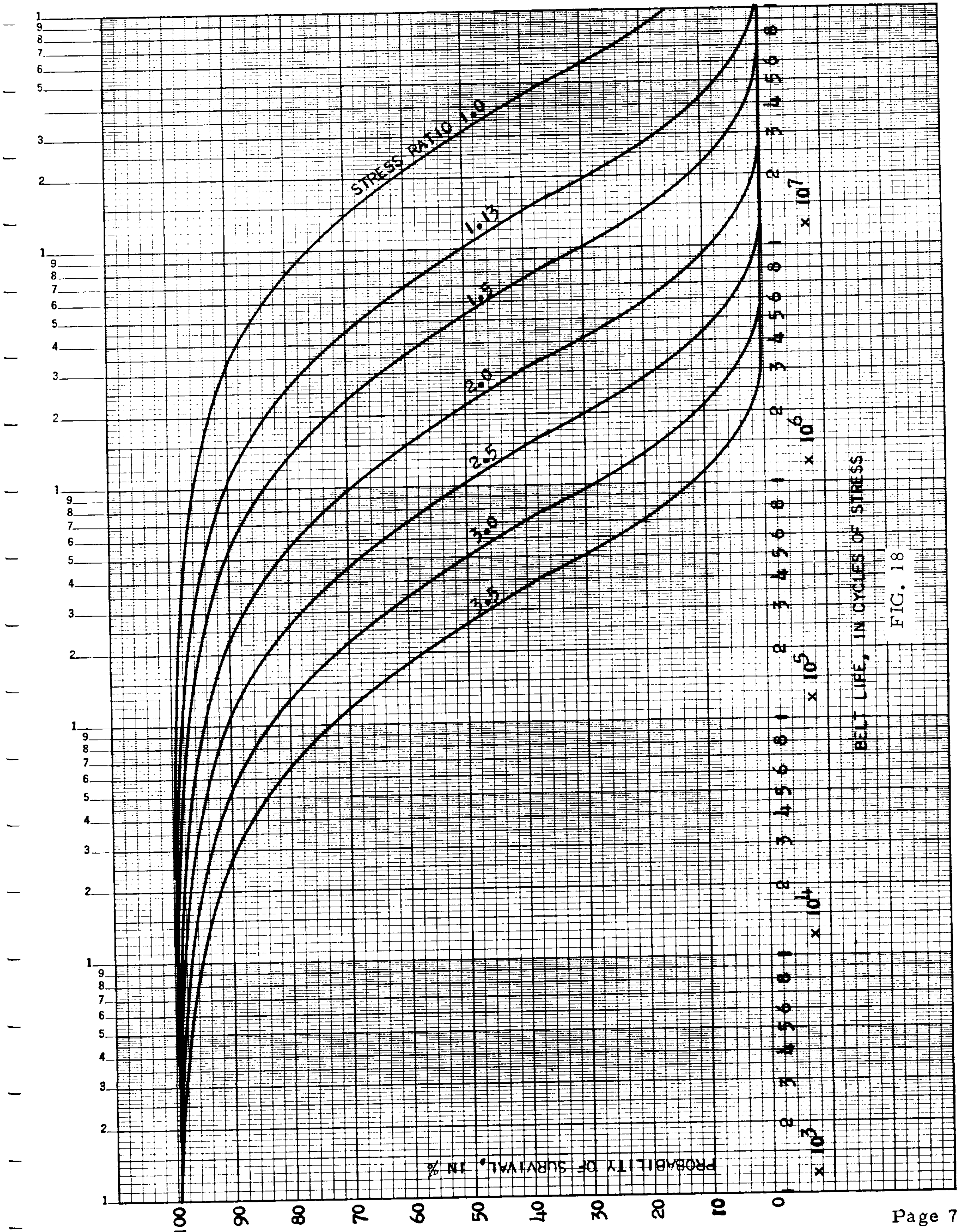

STRESS RATIO 1.0
1.13
1.5
2.0
2.5
3.0
3.5
BELT LIFE, IN CYCLES OF STRESS
PROBABILITY OF SURVIVAL, IN %
x 10^7
x 10^6
x 10^5
x 10^4
x 10^3

FIG. 18

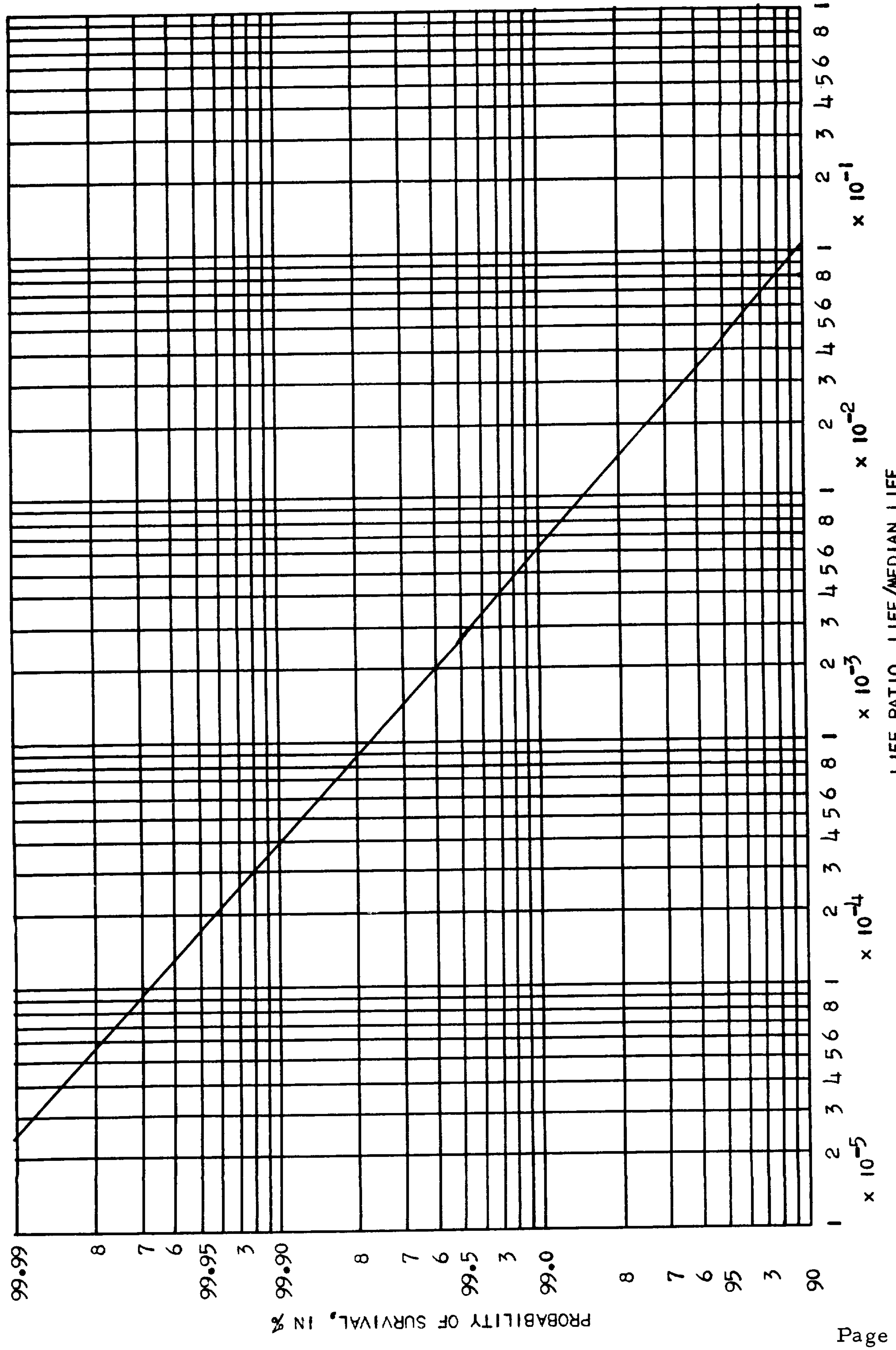

PROBABILITY OF SURVIVAL, IN %
99.99
8
7
6
99.95
3
99.90
8
7
6
99.5
3
99.0
8
7
6
95
3
90
LIFE RATIO, LIFE/MEDIAN LIFE
x 10^-5
x 10^-4
x 10^-3
x 10^-2
x 10^-1

Fig. 19

Date ____________

FATIGUE LIFE CALCULATION WORKSHEET

Name PLAYBACK DRIVE BELT Part No. ---

Belt Length 8.37 in.	Thickness (t) .001 in.	Width (w) .1875 in.
Installed stress S_o 2,500 psi	Torque: M = .0214 lb. in. -- oz. in. (Convert torque to lb. in. if given in oz. in.)	
DriverDiam. D_d .1875 in.	Speed 4,000 RPM	Required Life: ____ hrs.
Smallest Pulley Diam. D_s -- in. This data from design or application	Pulley Diam. D. ____ in. (Life Controlling)	

Stress Cycles per Belt Rev. = 1

Belt Revolutions per Hour $= \dfrac{3.14 \times D_d \times RPM \times 60}{L}$

$= \dfrac{188\,(.1875)\,(4{,}000)}{(8.37)} \times \dfrac{3.58^{*}}{32} = 1{,}880$ Rev/hr.

Stress Range = $S_{max} - S_{min}$ = (3,108) - (-2,108) = 5,216 psi

Minimum Stress S_{min} -2,108 psi

Endurance Limit = S_{el} 3,250 psi from Fig. 15 or 16

Stress Ratio $= \dfrac{\text{Stress Range}}{S_{el}} = \dfrac{(5{,}216)}{(3{,}250)} = 1.60$

Life = 1.3×10^6 cycles of stress from Fig. 17

Belt Life = 1.3×10^6 revolutions of belt

= 692 hours (~~Most Probable Value~~
(For lot of 2 at 95% confidence level.

______ % Probability of Survival for at least ______ hours of operation

Calculate S_{max} and S_{min} on reverse side

* Duty cycle correction in this application

Figure 20

Change in stress due to transmitted torque:

$$S_T = \frac{M}{D_d \quad tw} = \frac{(\ .0214\ )}{(\ .1875\ )(\ .001\ )(\ .1875\ )} = \underline{\quad 608 \quad} \text{psi}$$

$E = 7.5 \times 10^5$ for Polyester)
) USE NO OTHER VALUES
$= 5.9 \times 10^5$ for Polyimide)

Bending stress at Smallest Pulley on Life controlling side:

$$S_D = \frac{E \quad t}{D} = \frac{(7.5 \times 10^5)(\ .001\ )}{(\ .1875\ )} = \underline{\quad 4,000 \quad} \text{psi}$$

Bending stress at Smallest Pulley in Serpentine System (on opposite side of belt):

$$S_s = \frac{E \quad t}{D_s} = \frac{(\quad \times 10^5)(\quad)}{(\quad)} = \underline{\quad --- \quad} \text{psi}$$

Max Stress Outside Pattern:

S_o	=	2,500
Plus S_T	=	608
S_{max}	=	3,108

Minimum Stress All Patterns:

S_o	=	2,500
Minus S_T	=	608
Sub-total	=	1,892
Minus S_D	=	4,000
S_{min}	=	- 2,108

Max Stress Serpentine Pattern:*

S_o	=	
Plus S_T	=	
Sub-total	=	
Plus S_s	=	
S_{max}	=	

* Note: Base Calculations on side of belt which is in contact with pulleys. If both sides contact pulleys (with reversed bending) use the side of the belt which results in the algebraic highest value of S_{min}.

Figure 21

Date --

FATIGUE LIFE CALCULATION WORKSHEET

Name SERPENTINE BELT   Part No. --

Belt Length 37.5 in.	Thickness (t) .0015 in.	Width (w) .375 in.
Installed stress S_o 2,940 psi	Torque: M = .156 lb. in. ______ oz. in. (Convert torque to lb. in. if given in oz. in.)	
Driver Diam. D_d .3125 in.	Speed 120 RPM	Required Life: 8,800 hrs.
Smallest Pulley Diam. D_s .3125 in. This data from design or application	Pulley Diam. D. .625 in. (Life Controlling)	

Stress Cycles per Belt Rev. = 2

Belt Revolutions per Hour $= \dfrac{3.14 \times D_d \times RPM \times 60}{L}$

$= \dfrac{188\ (.3125)\ (120)}{(37.5)}$ x ______ = 188 Rev/hr.

Stress Range = $S_{max} - S_{min}$ = (7,430) - (+250) = 7,180 psi

Minimum Stress S_{min} +250 psi

Endurance Limit = S_{el} 2,500 psi from Fig. 15 or 16

Stress Ratio = $\dfrac{\text{Stress Range}}{S_{el}}$ $\dfrac{(7,180)}{(2,500)}$ = 2.87

Life = 2.0×10^5 cycles of stress from Fig. 17

Belt Life = 1.0×10^5 revolutions of belt

= 532 hours (~~Most Probable Value~~
(For lot of 2 at 95% confidence level.

______ % Probability of Survival for at least ______ hours of operation

Calculate S_{max} and S_{min} on reverse side

Figure 22

Change in stress due to transmitted torque:

$$S_T = \frac{M}{D_d \quad tw} = \frac{(\ .156\ )}{(\ .3125\ )\ (\ .0015)\ (\ .375)} = \underline{\quad 890 \quad} \text{ psi}$$

$$E = 7.5 \times 10^5 \text{ for Polyester}$$

$$= 5.9 \times 10^5 \text{ for Polyimide}$$

USE NO OTHER VALUES

Bending stress at Smallest Pulley on Life Controlling Side:

$$S_D = \frac{E \quad t}{D} = \frac{(\ 7.5 \times 10^5)\ (\ .0015\ )}{(\ .625\ )} = \underline{\quad 1,800 \quad} \text{ psi}$$

Bending stress at Smallest Pulley in Serpentine System (on opposite side of belt):

$$S_s = \frac{E \quad t}{D_s} = \frac{(\ 7.5 \times 10^5)\ (\ .0015\ )}{(\ .3125\ )} = \underline{\quad 3,600 \quad} \text{ psi}$$

Max. Stress Outside Pattern:

S_o =	
Plus S_T =	
S_{max} =	

Max. Stress Serpentine Pattern:*

S_o =	2,940
Plus S_T =	890
Sub-total =	[illegible],830
Plus S_s =	3,600
S_{max} =	7,430

Minimum Stress All Patterns:

S_o =	2,940
Minus S_T =	890
Sub-total =	2,050
Minus S_D =	1,800
S_{min} =	+ 250

* Note: Base Calculations on side of belt which is in contact with pulleys. If both sides contact pulleys (with reversed bending) use the side of the belt which results in the algebraic highest value of S_{min}.

Figure 23

APPENDIX D

Procedure for Laying Out Weibull Distribution Probability Paper

There are two basic techniques which can be used to lay out the portion failed (cumulative probability) scale. The first is by measurement and the second is a graphical technique. In any case, the size of the probability scale has a fixed relationship to the length of the logarithmic scale cycle. The values to be used in these two methods are derived from Table 2 in Reference 7.

The first method utilizes the first and fourth columns in Table 5. The fourth column provides the position of the corresponding portion failed (in column one). Select the range of values to be covered and lay out the Weibull Distribution scale on the abscissa with the logarithmic scale on the ordinate. The origin should be located about two-thirds from the left-hand margin. This arrangement of scales permits the direct use of the method of least squares. The opposite arrangement of scales would require the interchanging of subscripts in the forms with greater chance of error in the transposition of subscripts.

The second method uses the first and fifth column in Table 5. Plot a three-cycle log scale on the ordinate (semi-logarithmic paper may be used). Draw a vertical line in a position about two-thirds from the left-hand margin. Draw a line at 45° through the intersection of this vertical line and the end of the second log cycle from the bottom, with a positive slope (from the lower left to the upper right). The vertical line is the 50% failed point. Select any other percent failed value and read the corresponding number in the fifth column. The intersection of the number from the fifth column read on the log scale with the 45^{o} line is the location of the corresponding percent failed value. Draw a vertical line through this intersection and label.

Table 5 was derived from Table 2 Reference 7 in the following manner: The values of ϕ is subtracted from 1 to change the maxima in the reference into minima. The second column is taken directly from the column headed "y". This is the number of standard deviations to the value of percent failed from the mean of the function. The third column is a shift of the origin from the mean to the median of the function. The fourth column is obtained by multiplying the third column by 0.43428, a scale change to make a standard deviation equal in length to a log cycle. This makes the slope of the cumulative plot equal to the standard deviation of the distribution. The fifth column is the common antilogarithm of the fourth column. The value in the fifth column can also be found in Table 4f Reference 2. This method of derivation is given so that additional points may be obtained by recourse to Reference 7.

TABLE 5

Plotting Weibull Distribution Probability Paper

Portion Failed	Sigma	Sigma +0.3665	X in Log Cycle Base Lengths	Reading On Log Scle
0.001	-6.9073	-6.5408	-2.8405	0.00144
0.005	-5.2958	-4.9293	-2.1407	0.00722
0.01	-4.6002	-4.2337	-1.8386	0.0145
0.05	-2.9702	-2.6037	-1.1307	0.0740
0.10	-2.2500	-1.8835	-0.8180	0.1521
0.15	-1.8170	-1.4505	-0.6299	0.2345
0.20	-1.5000	-1.1335	-0.4922	0.3220
0.25	-1.2459	-0.8749	-0.3819	0.4150
0.30	-1.0309	-0.6644	-0.2885	0.5146
0.35	-0.8422	-0.4757	-0.2066	0.6214
0.40	-0.6717	-0.3052	-0.1325	0.7371
0.45	-0.5144	-0.1479	-0.0642	0.8626
0.50	-0.3665	0.0000	0.0000	1.0000
0.55	-0.2250	0.1415	0.0614	1.152
0.60	-0.0874	0.2791	0.1212	1.324
0.65	0.0486	0.4151	0.1803	1.515
0.70	0.1856	0.5521	0.2398	1.737
0.75	0.3266	0.6931	0.3010	2.000
0.80	0.4759	0.8424	0.3656	2.321
0.85	0.6403	1.0068	0.4372	2.737
0.90	0.8340	1.2005	0.5214	3.322
0.95	1.0972	1.4637	0.6356	4.321
0.99	1.5272	1.8937	0.8224	6.644
0.995	1.6674	2.0339	0.8832	7.642
0.999	1.9326	2.2991	0.9984	9.963